Linear transformations and the discrete uniform distribution

AS Further Statistics 1 | Edexcel 8FM0 | 12 questions, 71 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 71**

1 The random variable X has $E(X) = 4$ and $Var(X) = 2.5$

Find

a. $E(3X - 2)$

b. $Var(3X - 2)$

c. $Var(5 - 2X)$

(3 marks)

Answer _____

2 A fair twelve-sided dice has faces numbered 1 to 12. The random variable X is the score when the dice is rolled once.

a. Write down the name of the distribution of X.

b. Find $E(X)$ and $Var(X)$.

c. Find $P(X > 8)$.

(4 marks)

Answer _____

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3 The discrete random variable X has the probability distribution shown.

x	1	2	3
$P(X = x)$	0.3	0.5	0.2

a. Find E(X) and Var(X).

The random variable $Y = 2X + 3$

b. Find E(Y) and Var(Y).

(5 marks)

Answer

4 An orchard in Kent grades its apples by mass. The mass of an apple, M grams, is coded using

$$Y = \frac{M - 150}{5}$$

For the apples in one crate, E(Y) = 2.4 and Var(Y) = 1.44

Find the mean and the standard deviation of the mass of an apple in this crate.

(4 marks)

Answer

5 The random variable X has a discrete uniform distribution over the integers 1, 2, 3, ..., n

Given that Var(X) = 10

a. find the value of n

b. find E(X)

c. find P($X \leq 4$).

(5 marks)

Answer

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6 A fair spinner has ten equal sections labelled 3, 5, 7, 9, 11, 13, 15, 17, 19 and 21. The random variable S is the number the spinner lands on.

a. By writing $S = 2X + 1$, where X has a discrete uniform distribution over 1, 2, ..., 10, find E(S) and Var(S).

b. Find P($|S - \mu| < \sigma$), where μ and σ are the mean and standard deviation of S.

(6 marks)

Answer

7 The random variable X has a discrete uniform distribution over the integers 1 to 9

a. Find E(X) and show that Var(X) = $\frac{20}{3}$

The random variable $Y = aX + b$, where a and b are constants and $a > 0$. Given that E(Y) = 12 and Var(Y) = 60

b. find the value of a and the value of b.

(6 marks)

Answer

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8 At a charity stall in Leeds, a player pays £1 and rolls a fair six-sided dice once. If the dice shows X, the player wins $20X$ pence.

a. Find the expected amount the player wins.

b. Find the expected profit the stall makes on each game.

c. Find the standard deviation of the amount the player wins.

d. The charge for a game is changed to c pence. Find the smallest whole number c for which the expected profit on a game is at least 40% of the charge.

(7 marks)

Answer

9 Raffle tickets are numbered 1, 2, 3, ..., n. One ticket is drawn at random and its number is X. Given that $P(X \leq 5) = 0.25$

a. find the value of n

b. find $E(X)$ and $Var(X)$.

The random variable $Y = 4X - 2$

c. Find $E(Y)$ and $Var(Y)$.

(6 marks)

Answer

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10 The discrete random variable X has the probability distribution shown, where a and b are constants.

x	0	1	2	3
$\mathbf{P}(X = x)$	0.1	a	b	0.3

Given that $E(2X + 1) = 5$

a. find the value of a and the value of b

b. find $\text{Var}(5 - 3X)$

c. find $E(5 - 3X)$.

(7 marks)

Answer

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11 The random variable X has a discrete uniform distribution over the integers 1, 2, ..., n

a. Using $\Sigma r = \frac{1}{2}n(n + 1)$ and $\Sigma r^2 = \frac{1}{6}n(n + 1)(2n + 1)$, prove that $E(X) = \frac{n + 1}{2}$ and $Var(X) = \frac{n^2 - 1}{12}$

A school's sports day raffle sells tickets numbered 11 to 30 inclusive. One ticket is drawn at random and its number is T.

b. Find $E(T)$ and $Var(T)$.

c. Find the probability that the number on the ticket is a multiple of 4

(9 marks)

Answer

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12 A fair spinner is numbered 1, 2, 3, ..., n. The number it lands on is X. In a game at a school fete, a player's score is $S = 3X - 2$

a. Given that $E(S) = 13$, show that $n = 9$

b. Find $Var(S)$.

A player pays 75p for one spin and receives $5S$ pence.

c. Find the profit the fete should expect to make from 300 spins.

d. Find the standard deviation of the amount a player receives.

e. Find the probability that a player receives more than they paid.

(9 marks)

Answer

End of questions | Total 71 marks

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