Hypothesis tests for the geometric distribution

A-level Further Statistics 1 | Edexcel 9FM0 | 12 questions, 75 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 75**

1 In each situation X is the number of trials up to and including the first success. Write down suitable null and alternative hypotheses in terms of p, the probability of a success, and state whether small or large values of X would support the alternative hypothesis.

a. Jack thinks his dice shows a six less often than a fair dice would. He rolls it until he gets a six.

b. An office receives spam emails, and 5% of them used to get past the spam filter. After an update, the IT manager believes this proportion has increased. She counts the spam emails until one gets past the filter.

c. A machine used to jam on 30% of its runs. After a service, an engineer wants to test whether this proportion has changed. He counts the runs until the machine jams.

(3 marks)

Answer _____

2 A puzzle app claims that 25% of its daily puzzles are rated 'easy'. Daniel thinks that the proportion is lower than this. He opens puzzles one at a time and the first easy puzzle is the 12th one he opens.

Test, at the 5% level of significance, whether there is evidence to support Daniel's belief. State your hypotheses clearly.

(4 marks)

Answer _____

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3 Of a certain type of houseplant, 5% grow with variegated leaves. Cerys believes that a new compost increases this proportion. She examines plants grown in the new compost one at a time, and the first plant with variegated leaves is the 3rd plant she examines.

Test, at the 10% level of significance, whether there is evidence to support Cerys's belief. State your hypotheses clearly.

(4 marks)

Answer

4 At a café, 15% of customers usually buy a pastry. After a price rise, the manager wants to test whether this proportion has fallen. She will count the customers, one at a time, until the first customer who buys a pastry.

a. Write down suitable hypotheses for the test.

b. Find the critical region for the test at the 5% level of significance.

c. Find the actual significance level of the test.

(5 marks)

Answer

5 A courier company finds that 1% of the parcels it delivers are damaged. After moving to a new depot, a manager suspects that this proportion has increased. To test this, parcels from the new depot are inspected one at a time until a damaged parcel is found.

a. Find the critical region for a test at the 5% level of significance, stating your hypotheses clearly.

b. Find the actual significance level of the test.

(5 marks)

Answer

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6 On a website, 4% of visitors click on a banner advert. The page layout is changed and the web designer wants to test whether the proportion of visitors who click on the advert has changed. She will record the number of visitors, X, up to and including the first visitor who clicks on the advert.

a. Using a 10% level of significance, find the critical region for this test. The probability in each tail should be as close as possible to, but not more than, 5%

b. Find the actual significance level of the test.

(6 marks)

Answer

7 A coin-pusher machine at an amusement arcade is set so that 25% of plays give a payout. Ravi plays until he gets a payout, and records the number of plays, X.

a. Ravi wants to test, at the 5% level of significance, whether the probability of a payout is greater than 0.25. Explain why this test can never lead to H_0 being rejected.

b. Find the smallest significance level at which a result of $X = 1$ would lead to H_0 being rejected in this test.

c. Ravi's friend Asha instead tests, at the 5% level, whether the probability of a payout is less than 0.25. Find the critical region for Asha's test and its actual significance level.

(6 marks)

Answer

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8 A spinner is designed so that it lands on red with probability $\frac{1}{5}$ Ines believes that it lands on red less often than this. She will spin it until it lands on red, and record the number of spins, X.

a. Find the critical region for a test of Ines's belief at the 1% level of significance, stating your hypotheses clearly.

b. Find the actual significance level of the test.

The spinner lands on red for the first time on the 19th spin.

c. State, with a reason, the conclusion of the test at the 1% level.

d. Determine whether the conclusion would be different at the 5% level.

(7 marks)

Answer

9 A toy company claims that 30% of its mystery boxes contain a rare figure. Tamsin and Harvey both want to test, at the 5% level of significance, whether the proportion is less than 30%.

- Tamsin will open 15 boxes and record the number, Y, that contain a rare figure.
- Harvey will open boxes one at a time until he finds a rare figure, and record the number of boxes, X, that he opens.

a. Find the critical region for Tamsin's test and its actual significance level.

b. Find the critical region for Harvey's test and its actual significance level.

c. Under the company's claim, find the expected number of boxes Harvey opens, and hence give one advantage of Harvey's test compared with Tamsin's.

(8 marks)

Answer

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10 In archery practice, each of Seren's arrows hits the gold with probability 0.3, independently. A round consists of 3 arrows, and a round is called **good** if at least 2 of the 3 arrows hit the gold.

a. Show that the probability that a round is good is 0.216

Seren changes to a new bow. Her coach suspects that the probability of a good round has decreased. Seren shoots rounds until she has a good round, and the coach records the number of rounds, R.

b. Find the critical region for a test of the coach's suspicion at the 5% level of significance, stating your hypotheses clearly.

c. Write down the actual significance level of the test.

d. Seren's first good round with the new bow is her 11th round. State the conclusion of the test in context.

(8 marks)

Answer

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11 Each of 25 students in a class is given a dice and asked to test whether it shows a six less often than a fair dice would. Each student rolls their dice until it shows a six, records the number of rolls, X, and carries out a test at the 5% level of significance.

a. Find the critical region for each student's test, stating the hypotheses.

b. Find the actual significance level of the test.

c. Given that all 25 dice are in fact fair, find the probability that at least 2 of the students reject H_0.

d. One student's first six comes on roll 20, and she says her dice is biased. Comment on her claim in the light of your answer to part (c).

(9 marks)

Answer

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12 The number of faults per day on a factory's packing line has a Poisson distribution with mean 1.6. A day with no faults is called a **clear** day.

a. Show that the probability that a day is clear is 0.2019, to 4 decimal places.

After new machinery is installed, the engineer suspects that the mean number of faults per day has increased. She records the number of days, T, up to and including the first clear day.

b. Write down a suitable distribution for T and state an assumption needed.

c. Explain why an increase in the mean number of faults per day would tend to make T larger, and write down suitable hypotheses in terms of p, the probability that a day is clear.

d. Find the critical region for the test at the 5% level of significance.

e. Find the actual significance level of the test.

f. The first clear day after the machinery is installed is the 17th day. State the conclusion of the test in context.

(10 marks)

Answer

End of questions | Total 75 marks

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