PGFs of sums and linear functions

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** $G(t) = (1/5)(t^3 + 2t^5 + 2t^9)$ **b)** 2/5 **c)** 4/5	4
2	**a)** $G(t) = e^{2(t - 1)}e^{5(t - 1)} = e^{7(t - 1)}$, the pgf of Po(7) **b)** 0.0521	4
3	**a)** $G(t) = (1/12)(1 + 3t + 4t^2 + 4t^3)$ **b)** 1/3 **c)** 2/3	4
4	**a)** k = 1/625 **b)** $G(t) = t^2(2 + 3t^3)^4/625$ **c)** 216/625 **d)** E(W) = 9.2	8
5	(a) $E(t^{aX + b}) = t^b E((t^a)^X)$, so the pgf of Y is $t^b G(t^a)$, where G is the pgf of X; (b) differentiating, the derivative at t = 1 is bG(1) + aG'(1) = b + aE(X); (c) pgf of $Y = t(3 + t^4)^2/16$, E(Y) = 3	6
6	**a)** $t/(3 - 2t)$ **b)** $t^3/(3 - 2t)^3$ **c)** Negative binomial, r = 3, p = 1/3 **d)** 8/81 **e)** E(S) = 9, Var(S) = 18	9
7	**a)** $G(t) = t(1 + t)^2/4$ shown **b)** $t(1 + t)^5/32$ **c)** 5/4 **d)** 5/16 **e)** B(5, 1/2)	11
8	**a)** 20 **b)** $G(t) = t^2/(5 - 4t)^2$: negative binomial, r = 2, p = 1/5 **c)** 0.7373	11
9	**a)** $X \sim Po(4)$ **b)** E(Y) = 11, Var(Y) = 16 **c)** 2X + 3 is always odd, so it can never be 8 **d)** $8e^{-4}$	9
10	**a)** $G(t) = t^2(3 + 2t + t^2)^2/36$ shown **b)** E(T) = 10/3, Var(T) = 10/9 **c)** $(9t^6 + 12t^11 + 10t^16 + 4t^21 + t^26)/36$ **d)** P(S = 16) = 5/18, P(S > 20) = 5/36 **e)** E(S) = 38/3, Var(S) = 250/9	10
11	**a)** $X + Y \sim Po(4)$; $(32/3)e^{-4} = 0.1954$ **b)** $G(t) = e^{1.5(t^2 - 1)} \times e^{2.5(t - 1)}$ shown **c)** E(Z) = 5.5, Var(Z) = 8.5 **d)** Mean 5.5 is not equal to variance 8.5 **e)** 0.0847	11
12	**a)** $G(t) = t(a + bt)^2/(3 - 2t)$ **b)** a = 1/2, b = 1/2 **c)** E(W) = 4, Var(W) = 13/2 **d)** 25/108 **e)** $t^4(1 + t^3)^2/(4(3 - 2t^3))$	12

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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