PGFs of sums and linear functions

A-level Further Statistics 1 | Edexcel 9FM0 | 12 questions, 99 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 99**

1 The discrete random variable X has probability generating function

$G_X(t) = \frac{1}{5}(1 + 2t + 2t^3)$

The random variable $Y = 2X + 3$

a. Find the probability generating function of Y.

b. Write down $P(Y = 9)$.

c. Find $P(Y > 4)$.

(4 marks)

Answer _____

2 The independent random variables X and Y are such that $X \sim Po(2)$ and $Y \sim Po(5)$.

a. Use probability generating functions to show that $X + Y \sim Po(7)$.

b. Find $P(X + Y = 3)$.

(4 marks)

Answer _____

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3 The independent discrete random variables X and Y have probability generating functions

$G_X(t) = \frac{1}{3}(1 + 2t)$ and $G_Y(t) = \frac{1}{4}(1 + t + 2t^2)$

a. Find the probability generating function of $S = X + Y$, giving your answer as a polynomial in t.

b. Find $P(S = 2)$.

c. Find $P(S \geq 2)$.

(4 marks)

Answer

4 The random variable X has probability generating function

$G_X(t) = k(2 + 3t)^4$

where k is a constant.

a. Find the value of k.

The random variable $W = 3X + 2$

b. Find the probability generating function of W.

c. Find the exact value of $P(W = 11)$.

d. Use calculus and your answer to (b) to find $E(W)$.

(8 marks)

Answer

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5 The discrete random variable X has probability generating function $G_X(t)$. The random variable $Y = aX + b$, where a and b are positive integers.

a. Show that $G_Y(t) = t^b G_X(t^a)$

b. Hence, by differentiating, show that $E(Y) = aE(X) + b$

The random variable X has probability generating function $G_X(t) = \frac{1}{16}(3 + t)^2$ and $Y = 4X + 1$

c. Write down $G_Y(t)$ and find $E(Y)$.

(6 marks)

Answer

6 Ali, Beth and Chen each roll their own fair six-sided dice repeatedly until they get a 5 or a 6. The random variables X_1, X_2 and X_3 are the numbers of rolls they make, and $S = X_1 + X_2 + X_3$

a. Write down the probability generating function of X_1.

b. Find the probability generating function of S, in its simplest form.

c. Hence name the distribution of S, including its parameters.

d. Find the exact value of $P(S = 5)$.

e. Find $E(S)$ and $Var(S)$.

(9 marks)

Answer

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7 The discrete random variable V has the probability distribution shown.

v	1	2	3
$\mathbf{P}(V = v)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

a. Show that the probability generating function of V is $G_V(t) = \frac{t(1 + t)^2}{4}$

The discrete random variable W is independent of V and has probability generating function $G_W(t) = \frac{(1 + t)^3}{8}$

b. Find the probability generating function of $U = V + W$ in its simplest form.

c. Use calculus to find $\text{Var}(U)$.

d. Find $P(U = 4)$.

e. State the distribution of $U - 1$, giving its parameters.

(11 marks)

Answer

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8 The random variable X has probability generating function

$$G_X(t) = \frac{1}{5 - 4t}$$

a. Use calculus to find Var(X).

The independent random variables X_1 and X_2 each have the same distribution as X, and $Y = X_1 + X_2 + 2$

b. By finding the probability generating function of Y, name the distribution of Y and give its parameters.

c. Hence find $P(X_1 + X_2 > 3)$.

(11 marks)

Answer

9 The discrete random variable Y has probability generating function

$G_Y(t) = t^3e^{4(t^2 - 1)}$

a. Show that $Y = 2X + 3$, where X has a Poisson distribution with a mean that you should state.

b. Use calculus to find $E(Y)$ and $Var(Y)$.

c. Explain why $P(Y = 8) = 0$

d. Find the exact value of $P(Y = 7)$.

(9 marks)

Answer

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10 A spinner at a charity fun day scores 1, 2 or 3 with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively.

Priya spins it twice, and the scores are independent. T is her total score.

a. Show that the probability generating function of T is $G_T(t) = \frac{t^2(3 + 2t + t^2)^2}{36}$

b. Use calculus to find E(T) and Var(T).

Priya is awarded $S = 5T - 4$ points.

c. Find the probability generating function of S, as a polynomial.

d. Find P($S = 16$) and P($S > 20$).

e. Write down E(S) and Var(S).

(10 marks)

Answer

11 An office manager, Tomasz, receives urgent emails at a mean rate of 1.5 per hour and routine emails at a mean rate of 2.5 per hour, independently and at random. In a randomly chosen hour, X is the number of urgent emails and Y is the number of routine emails.

a. Use probability generating functions to find the distribution of $X + Y$, and hence find the exact probability that Tomasz receives 3 emails in a randomly chosen hour.

Urgent emails count 2 points each and routine emails 1 point each towards a workload score $Z = 2X + Y$.

b. Show that the probability generating function of Z is $G_Z(t) = e^{1.5t^2 + 2.5t - 4}$

c. Use calculus to find $E(Z)$ and $Var(Z)$.

d. Explain why Z does not have a Poisson distribution.

e. Find $P(Z = 2)$, giving your answer to 4 decimal places.

(11 marks)

Answer

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12 The independent random variables X and Y have probability generating functions

$G_X(t) = (a + bt)^2$ and $G_Y(t) = \frac{t}{3 - 2t}$

where a and b are positive constants. The random variable $W = X + Y$

a. Write down the probability generating function of W.

b. Given that $P(W = 1) = \frac{1}{12}$, find the values of a and b.

c. Use calculus to find $E(W)$ and $Var(W)$.

d. Find the exact value of $P(W = 3)$.

e. Find the probability generating function of $3W + 1$

(12 marks)

Answer _____

End of questions | Total 99 marks

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