Probability generating functions

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** $G(t) = 1/8 + (3/8)t + (1/4)t^2 + (1/4)t^3$ **b)** $E(X) = 13/8$	4
2	**a)** $k = 1/7$ **b)** 2/7 **c)** 6/7 **d)** 0 (no t^3 term)	4
3	**a)** $(0.7 + 0.3t)^8$ **b)** $e^{2.5(t - 1)}$ **c)** $t/(4 - 3t)$ **d)** Po(3) **e)** B(5, 1/3)	5
4	**a)** $k = 1/64$ shown **b)** 27/64 **c)** $Var(X) = 9/4$ shown	9
5	**a)** $G''(1) = 3$ shown **b)** $a = 0.3$, $b = 0.3$, $c = 0.4$	7
6	**a)** $k = 1/\ln 3$ **b)** $2/\ln 3$ **c)** $6/\ln 3 - 4/(\ln 3)^2$ **d)** $8/(81 \ln 3)$	10
7	(a) $G(t) = \Sigma\, p\, q^{x-1}\, t^x = pt\, \Sigma\, (qt)^{x-1} = pt/(1 - qt)$, $q = 1 - p$, a geometric series with $\|qt\| < 1$; (b) $G'(t) = p/(1 - qt)^2$ gives $E(X) = 1/p$; $G''(t) = 2pq/(1 - qt)^3$ gives $G''(1) = 2q/p^2$ and $Var(X) = 2q/p^2 + 1/p - 1/p^2 = q/p^2 = (1 - p)/p^2$	8
8	**a)** Negative binomial with $r = 3$, $p = 1/4$ **b)** Number of spins up to and including the third red **c)** $E(X) = 12$, $Var(X) = 36$ **d)** 27/512	11
9	**a)** $G(t) = 1/(2 - t)^2$ shown **b)** $E(X) = 2$, $Var(X) = 4$	8
10	**a)** $E(X) = np$, $Var(X) = npq$ shown **b)** $n = 8$, $p = 0.6$ **c)** 0.2090	10
11	**a)** $G(t) = (6t + 5t^2 + 25t^3)/36$ shown **b)** $E(R) = 91/36$, $Var(R) = 755/1296$ **c)** 50.6p (about 51p)	9
12	**a)** $G(1) = 1$ so $(a + b + c)^2 = 1$, and $a + b + c > 0$ **b)** $a = 1/4$, $b = 1/4$, $c = 1/2$ **c)** 5/16 **d)** $Var(X) = 11/8$	12

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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