Probability generating functions

A-level Further Statistics 1 | Edexcel 9FM0 | 12 questions, 97 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 97**

1 The discrete random variable X has the probability distribution shown.

x	0	1	2	3
$\mathbf{P}(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

a. Write down the probability generating function of X.

b. Use calculus to find $\mathrm{E}(X)$.

(4 marks)

Answer _____

2 The discrete random variable X has probability generating function

$G_X(t) = k(t + 2t^2 + 4t^4)$

where k is a constant.

a. Find the value of k.

b. Write down $\mathrm{P}(X = 2)$.

c. Find $\mathrm{P}(X \geq 2)$.

d. Write down $\mathrm{P}(X = 3)$, giving a reason.

(4 marks)

Answer _____

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3 Write down the probability generating function of each of these random variables.

a. $A \sim \text{B}(8, 0.3)$

b. $B \sim \text{Po}(2.5)$

c. $C \sim \text{Geo}(\frac{1}{4})$

Each of these is the probability generating function of a standard distribution. Name the distribution, including its parameters.

d. $G_D(t) = e^{3t - 3}$

e. $G_V(t) = \frac{(2 + t)^5}{243}$

(5 marks)

Answer

4 The discrete random variable X has probability generating function

$G_X(t) = k(1 + 3t^2)^3$

where k is a constant.

a. Show that $k = \frac{1}{64}$

b. Find $P(X = 4)$.

c. Use calculus to show that $\text{Var}(X) = \frac{9}{4}$

(9 marks)

Answer

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5 The discrete random variable X takes only the values 1, 2 and 3, and has probability generating function

$G_X(t) = at + bt^2 + ct^3$

where a, b and c are constants. Given that $E(X) = 2.1$ and $Var(X) = 0.69$,

a. show that $G_X''(1) = 3$

b. find the values of a, b and c.

(7 marks)

Answer

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6 The discrete random variable X has probability generating function

$$G_X(t) = k \ln\left(\frac{3}{3 - 2t}\right)$$

where k is a constant.

a. Find the exact value of k.

b. Find the exact value of $E(X)$.

c. Find the exact value of $\text{Var}(X)$.

d. Find $P(X = 3)$, giving your answer in the form $\frac{p}{q \ln 3}$ where p and q are integers.

(10 marks)

Answer

7 The random variable $X \sim \text{Geo}(p)$, so that $P(X = x) = p(1 - p)^{x - 1}$ for $x = 1, 2, 3, \ldots$

a. Show that the probability generating function of X is $G_X(t) = \frac{pt}{1 - (1 - p)t}$

b. Use calculus to show that $E(X) = \frac{1}{p}$ and $\text{Var}(X) = \frac{1 - p}{p^2}$

(8 marks)

Answer

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8 The discrete random variable X has probability generating function

$$G_X(t) = \frac{t^3}{(4 - 3t)^3}$$

a. Specify fully the distribution of X.

A spinner has four equal sectors, one of which is red. The spinner is spun repeatedly.

b. Describe an outcome that could be modelled by X.

c. Use calculus to find $E(X)$ and $Var(X)$.

d. Find the exact value of $P(X = 5)$.

(11 marks)

Answer

9 The discrete random variable X has probability function

$$P(X = r) = \frac{r + 1}{2^{r+2}} \quad r = 0, 1, 2, \ldots$$

a. Show that the probability generating function of X is $G_X(t) = \frac{1}{(2 - t)^2}$

b. Use calculus to find $E(X)$ and $Var(X)$.

(8 marks)

Answer

10 The random variable $X \sim B(n, p)$ has probability generating function $G_X(t) = (q + pt)^n$, where $q = 1 - p$.

a. Use calculus to show that $E(X) = np$ and $Var(X) = npq$.

A bakery puts n biscuits in each tin. Each biscuit, independently, has probability p of being broken. The number of broken biscuits in a tin has mean 4.8 and variance 1.92.

b. Find the values of n and p.

c. Find the probability that a randomly chosen tin contains exactly 6 broken biscuits.

(10 marks)

Answer

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11 In a game at a school fair, Bilal rolls a fair six-sided dice until he gets a six, but he is allowed at most 3 rolls. The random variable R is the number of rolls he makes.

a. Show that the probability generating function of R is $G_R(t) = \frac{1}{36}(6t + 5t^2 + 25t^3)$

b. Use calculus to find the exact values of $E(R)$ and $Var(R)$.

Each roll costs 20p.

c. Find the expected amount Bilal pays to play the game.

(9 marks)

Answer

12 The discrete random variable X has probability generating function

$G_X(t) = (a + bt + ct^2)^2$

where a, b and c are positive constants.

a. Explain why $a + b + c = 1$

Given that $P(X = 0) = \frac{1}{16}$ and $E(X) = 2.5$,

b. find the values of a, b and c

c. find $P(X = 2)$

d. use calculus to find $\text{Var}(X)$.

(12 marks)

Answer

End of questions | Total 97 marks

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