Size and power of a test

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** 0.0193 **b)** 0.2528	4
2	**a)** $X \geq 7$ **b)** 0.0335 **c)** 0.2378	5
3	**a)** $c = 2$, size 0.0975 **b)** $1 - (1 - p)^2$ **c)** 0.36	5
4	**a)** $X \leq 2$ **b)** $(1 - p)^18[(1 - p)^2 + 20p(1 - p) + 190p^2] = (1 - p)^18(1 + 18p + 171p^2)$ **c)** 0.6769	6
5	**a)** $X \leq 1$ **b)** $P(X \leq 1) = e^(-\lambda) + \lambda e^(-\lambda) = e^(-\lambda)(1 + \lambda)$ **c)** 0.4060 and 0.0916 **d)** A smooth decreasing curve from (0, 1) through the plotted values to (5, 0.0404) **e)** 0.0404	7
6	**a)** $\bar{X} > 51.64$ **b)** 0.05 **c)** 0.8038 **d)** Increase: $4/\sqrt{n}$ is smaller, so $\bar{X}$ is more concentrated about 52.5 and more likely to be in the critical region	6
7	**a)** $X \leq 3$, size 0.0332 **b)** $Y \geq 10$, size 0.0404 **c)** Test A 0.7636, Test B 0.3874 **d)** Test A: smaller size (less chance of a Type I error) and much higher power (less chance of a Type II error)	9
8	**a)** $c = 200 - 2.3263 \times 12/\sqrt{n} = 200 - 27.916/\sqrt{n}$ **b)** 75 **c)** 196.78	8
9	**a)** 0.0722 **b)** 0.7575 **c)** 23.8 **d)** Test P: similar size and only slightly lower power, but fewer bulbs inspected on average (23.8 against 30)	9
10	**a)** A: $X \geq 6$, size 0.0166; B: $Y \geq 9$, size 0.0214 **b)** $1 - e^(-\lambda)(1 + \lambda + \lambda^2/2 + \lambda^3/6 + \lambda^4/24 + \lambda^5/120)$ **c)** A: 0.1424, B: 0.2709 **d)** Test B: its size is still below 0.05 and its power is much higher	9
11	**a)** Mean $(5 + 1)/2 = 3$, variance $(25 - 1)/12 = 2$ **b)** Size $= 2 \times P(Z < -1.6432) = 0.100$ **c)** 0.7130 **d)** Power increases: $\text{Var}(\bar{X}) = \sigma^2/n$ falls, so $\bar{X}$ is more concentrated about 3.4 and more likely to exceed 3.3	10
12	**a)** $P(X \leq 1) = 0.1 > 0.05$, so even $X = 1$ is not significant: no critical region **b)** $Y \leq 3$, size 0.028 **c)** $P(Y \leq 3) = p^2 + 2p^2(1 - p) = p^2(3 - 2p)$ **d)** Increasing curve from (0, 0) to (1, 1), through (0.5, 0.5) **e)** $0.5^2(3 - 2 \times 0.5) = 0.5$	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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