Type I and type II errors

A-level Further Statistics 1 | Edexcel 9FM0 | 12 questions, 79 marks

Further Maths topic practice

Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ **/ 79**

1 A factory has found that 6% of the phone cases it makes are faulty. After a new moulding machine is installed, the manager, Gareth, tests whether the proportion of faulty cases has increased, using H_0: $p = 0.06$ and H_1: $p > 0.06$

a. Describe, in context, what is meant by a Type I error in this test.

b. Describe, in context, what is meant by a Type II error in this test.

c. Gareth's test does not reject H_0. State which type of error he could have made.

(3 marks)

Answer _____

2 A seed company claims that 35% of its sweet pea seeds produce white flowers. Hannah thinks the proportion is lower. She plants 20 seeds and tests H_0: $p = 0.35$ against H_1: $p < 0.35$ at the 5% level of significance. The critical region is $X \leq 3$, where X is the number of seeds that produce white flowers.

a. Find P(Type I error) for this test.

b. Explain why P(Type I error) is not equal to 0.05

(3 marks)

Answer _____

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

3 The number of calls to a helpline between 9 am and 10 am has a Poisson distribution with mean 4. After an advert is shown on television, the manager wants to test whether the mean number of calls in this hour has increased.

a. Using a 5% level of significance, find the critical region for the test. State your hypotheses clearly.

b. Write down P(Type I error) for this test.

(4 marks)

Answer

4 Past records show that 20% of visitors to a museum buy a guidebook. The museum redesigns the guidebook and wants to test whether this proportion has increased. A random sample of 25 visitors is taken and the number who buy a guidebook, X, is recorded.

a. Find the critical region for a test of H_0: $p = 0.2$ against H_1: $p > 0.2$ at the 5% level of significance.

b. Given that the true proportion is now 0.3, find P(Type II error).

(5 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

5 A school receives emails from parents at a mean rate of 8 per day. After a new website is launched, the head teacher wants to test, at the 5% level of significance, whether the mean number of emails per day has changed, using the number of emails received on one randomly chosen day.

a. Find the critical region for the test. The probability in each tail should be less than 0.025 and as close to 0.025 as possible. State the probability in each tail.

b. Find P(Type I error) for this test.

c. Given that the mean number of emails per day is in fact 12, find P(Type II error).

(7 marks)

Answer

6 A game developer states that the probability of finding a rare item in a loot box is 0.2. Zara believes the probability is lower. She opens loot boxes one at a time until she finds a rare item, and records the number of boxes opened, X, including the one containing the rare item.

a. Using a 5% level of significance, find the critical region for a test of H_0: $p = 0.2$ against H_1: $p < 0.2$

b. Write down P(Type I error).

c. Given that the probability of finding a rare item is actually 0.1, find P(Type II error).

(6 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

7 Cyclists pass an automatic counter on a cycle path at a mean rate of 3 every 10 minutes. Roadworks start nearby, and in a randomly chosen 40-minute period the counter records 6 cyclists.

a. Test, at the 5% level of significance, whether there is evidence of a decrease in the rate at which cyclists pass the counter. State your hypotheses clearly.

A further randomly chosen 20-minute period is observed and the test is repeated, again at the 5% level.

b. Find the critical region for this test.

c. Given that the true rate is now 1.5 cyclists every 10 minutes, find the probability of a Type II error.

(9 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

8 The mass of honey in a jar is normally distributed with mean μ grams and standard deviation 8 grams. The label says that μ = 250. A trading standards officer suspects that the jars are underfilled. She weighs a random sample of 16 jars and calculates the sample mean mass $\overline{X}$.

a. Using a 5% level of significance, find the critical region for $\overline{X}$. State your hypotheses clearly.

b. Given that μ is actually 245, find P(Type II error).

(7 marks)

Answer

9 A machine makes bolts whose lengths are normally distributed with mean μ mm and standard deviation 0.5 mm. To check that μ = 12, an engineer measures a random sample of 20 bolts and rejects H_0: μ = 12 in favour of H_1: μ ≠ 12 if the sample mean $\overline{X}$ satisfies $\overline{X}$ < 11.8 or $\overline{X}$ > 12.2

a. Find P(Type I error) for this test.

b. Given that μ = 12.3, find P(Type II error).

(6 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

10 A council receives complaints about missed bin collections at a mean rate of 4.5 per month. After a change of contractor, the council records the number of complaints in a series of 2-month periods and uses each period to test whether the mean number of complaints per month has changed.

a. Stating your hypotheses clearly and using a 5% level of significance, find the critical region for this test. The probability in each tail should be less than 0.025 and as close to 0.025 as possible.

b. State P(Type I error) for this test.

c. The test is carried out for each of the 6 two-month periods in a year. Given that the mean rate is in fact still 4.5 per month, find the probability that at least 2 of these tests give a significant result.

d. Given instead that the mean rate is actually 3 per month, find P(Type II error) for one test.

(10 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

11 The number of weeds in a 1 m^2 quadrat in a playing field has a Poisson distribution with mean 3. A groundsman uses a new treatment and believes it has reduced the mean number of weeds. He counts the weeds in 50 randomly placed quadrats and calculates the sample mean $\overline{X}$.

a. Using a suitable approximation and a 5% level of significance, find the critical region for $\overline{X}$ for a test of H_0: $\lambda = 3$ against H_1: $\lambda < 3$

b. Given that the mean number of weeds per quadrat is in fact 2.6, find P(Type II error).

c. The groundsman uses a 1% level of significance instead, with the same sample size. Without further calculation, state, with a reason, the effect on P(Type II error).

(9 marks)

Answer

12 A café owner, Mei, knows that 25% of her customers used to pay with cash. She wants to test whether this proportion has changed. She records how 30 randomly chosen customers pay.

a. Stating your hypotheses clearly, find the critical region for a test at the 5% level of significance. The probability in each tail should be less than 0.025 and as close to 0.025 as possible.

b. Find P(Type I error) for this test.

c. Given that the true proportion paying with cash is 0.45, find P(Type II error).

d. In Mei's sample, 13 customers pay with cash. State the conclusion of the test and state, with a reason, which type of error Mei might have made.

(10 marks)

Answer

End of questions | Total 79 marks

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.