Weekly check: Grade 7

Worked solutions

GCSE Maths weekly check

All weekly checks online

1 In triangle ABC, AB = 3 cm, AC = 5 cm and BC = √19 cm.
Find the size of angle BAC.

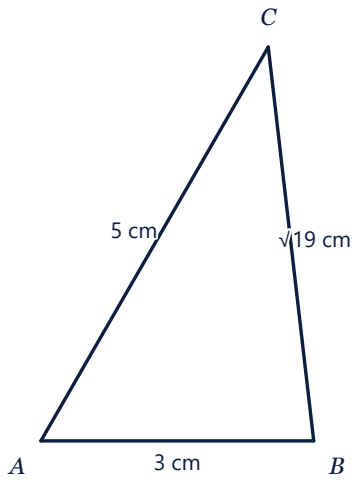

Diagram NOT accurately drawn

(4 marks)

Angle BAC is opposite BC, so use the cosine rule for the angle

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC}$$

$$\cos A = \frac{9 + 25 - 19}{2 \times 3 \times 5} = \frac{15}{30} = \frac{1}{2}$$

$\cos A = \frac{1}{2}$, so angle $BAC = 60^\circ$

$\boxed{60^\circ}$

2 Show that √12 + √27 can be written as 5√3.

(2 marks)

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

$$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$$

$$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

$\boxed{5\sqrt{3}}$

3 ABCD is a parallelogram. AB = 12 cm, AD = 7.5 cm and angle DAB = 65°.

Calculate the area of the parallelogram. Give your answer correct to 3 significant figures. (2 marks)

Parallelogram = 2 identical triangles

$$\text{Area} = 2 \times \frac{1}{2} \times 12 \times 7.5 \times \sin 65^\circ$$

$$\text{Area} = 12 \times 7.5 \times \sin 65^\circ$$

$$\text{Area} = 81.568...$$

$$81.6 \text{ cm}^2$$

4 A and B are two events.

$P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$

a. Work out $P(A \cap B)$.

b. Work out $P(B \mid A)$. (4 marks)

a)

$$0.7 = 0.5 + 0.4 - P(A \cap B)$$

$$P(A \cap B) = 0.9 - 0.7$$

$$P(A \cap B) = 0.2$$

b)

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B \mid A) = \frac{0.2}{0.5}$$

a) 0.2 b) 0.4

5 A, B and C are points on a circle with centre O.

B lies on the major arc AC.

Angle AOC = 128°.

Work out the size of angle ABC. Give a reason for your answer.

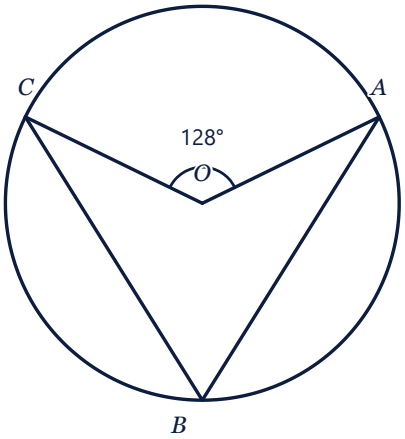

Diagram NOT accurately drawn

(3 marks)

The angle at the centre of a circle is twice the angle at the circumference when both stand on the same arc

Angle ABC = $128^\circ \div 2 = 64^\circ$

64°

6 The line $y = ax + 5$ is perpendicular to the line $by = 3x - 2$.

Find b in terms of a.

(3 marks)

$m_1 = a$

$y = \frac{3}{b}x - \frac{2}{b}$

$m_2 = \frac{3}{b}$

Perpendicular: product of gradients = -1

$a \times \frac{3}{b} = -1$

$\times b$ $\quad 3a = -b$ $\times b$

$b = -3a$

7 Work out the value of $(\frac{36}{25})^{\frac{1}{2}}$

(2 marks)

A power of $\frac{1}{2}$ means the square root of the numerator and of the denominator

$\sqrt{36} = 6$ and $\sqrt{25} = 5$, so the answer is $\frac{6}{5}$

$\boxed{\frac{6}{5}}$

8 Write $x^2 + 8x + 3$ in the form $(x + a)^2 + b$, where a and b are integers.

(2 marks)

Expanding $(x + a)^2$ gives $x^2 + 2ax + a^2$. To match $8x$, we need $2a = 8$, so $a = 4$

Halve the coefficient of x

$(x + 4)^2 = x^2 + 8x + 16$

Adjust the constant

$x^2 + 8x + 3 = (x + 4)^2 - 16 + 3 = (x + 4)^2 - 13$

Check by expanding

$x^2 + 8x + 16 - 13 = x^2 + 8x + 3$

$\boxed{(x + 4)^2 - 13}$

9 Two mathematically similar candles, C and D, are made from the same wax.

C has surface area 36 cm^2 and D has surface area 81 cm^2.

The volume of C is 24 cm^3.

Work out the volume of D.

(3 marks)

Area ratio C : D = 36 : 81 = 4 : 9

Length ratio = $\sqrt{4} : \sqrt{9} = 2 : 3$

Volume ratio = $2^3 : 3^3 = 8 : 27$

Volume of D = $24 \times 27 \div 8 = 81\ cm^3$

$\boxed{81\ cm^3}$

10 The frequency densities for the times, t minutes, that 80 customers waited in a post office queue are shown in the table.

Time (t minutes)	Frequency density
$0 < t \leq 10$	2
$10 < t \leq 30$	1.5
$30 < t \leq 40$	3

Mia says, "The $0 < t \leq 10$ class must have the most customers because its bar is tall." Is Mia correct? You must show your working.

(2 marks)

Frequency = frequency density x class width

0-10: 2 x 10 = 20

10-30: 1.5 x 20 = 30

30-40: 3 x 10 = 30

No - 0-10 has the fewest (20)

11 Factorise $2x^2 + 7x + 3$

(2 marks)

Look for two numbers that multiply to 2 x 3 = 6 and add to 7: they are 6 and 1

Split the middle term

$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3)$

Both groups now contain the whole factor $(x + 3)$. Taking out that shared factor leaves $2x + 1$, giving the two brackets below

Factorise: $(2x + 1)(x + 3)$

Check by expanding

$2x^2 + 6x + x + 3 = 2x^2 + 7x + 3$, correct

$$(2x + 1)(x + 3)$$

12 In triangle ABC, angle BAC = 52°, angle ACB = 63° and AC = 11 cm.

Calculate the length of BC. Give your answer correct to 3 significant figures.

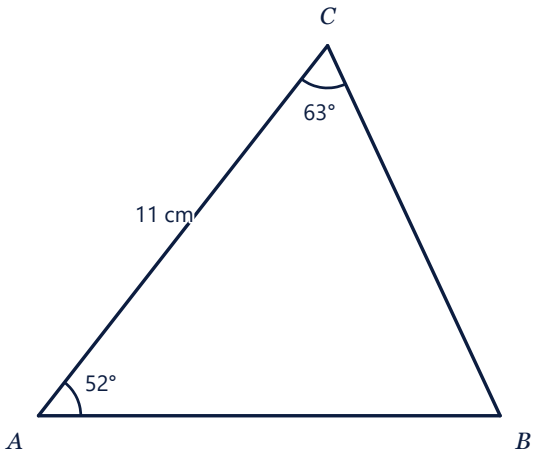

Diagram NOT accurately drawn

(3 marks)

Angle ABC = $180^\circ - 52^\circ - 63^\circ = 65^\circ$

BC is opposite angle A and AC is opposite angle B

Sine rule

$$\frac{BC}{\sin A} = \frac{AC}{\sin B}$$

$BC = 11 \times \sin 52^\circ \div \sin 65^\circ = 11 \times 0.78801... \div 0.90630..$

$BC = 9.564... = 9.56$ cm (3 sf)

9.56 cm

13 p is inversely proportional to q.

When $q = 5$, $p = 12$.

a. Find a formula for p in terms of q.

b. Find the value of p when $q = 8$.

(3 marks)

Inversely proportional means that the product pq is constant

$pq = k$. Dividing by q gives $p = k/q$

Substitute

$12 = \frac{k}{5}$, so $k = 60$ and $p = \frac{60}{q}$

When $q = 8$, $p = 60 \div 8 = 7.5$

14 A park keeper wants to estimate the number of fish in a lake.
She catches 30 fish, tags them and puts them back in the lake.
The next day she catches 40 fish. 8 of these fish are tagged.
Work out an estimate for the number of fish in the lake.

(2 marks)

Proportion of tagged fish in the second sample = $\frac{8}{40} = \frac{1}{5}$

Assume the same proportion of the whole population is tagged, so 30 fish is $\frac{1}{5}$ of the population

Estimate = $30 \times 40 \div 8 = 150$

150

15 $f(x) = 3x - 4$ $g(x) = x^2$

a. Find $f(5)$

b. Find $gf(2)$

(3 marks)

$gf(x)$ means $g(f(x))$: apply f first, then put its output into g. The function nearest the input is done first; gf does not mean multiply g by f

a) $f(5) = 3(5) - 4 = 11$

b) $f(2) = 3(2) - 4 = 2$, then $g(2) = 2^2 = 4$

16 Here are four equations.

A: $y = 3 \times 1.2^x$ B: $y = 5 \times 0.8^x$ C: $y = 0.5 \times 4^x$ D: $y = 2 \times (⅓)^x$

Which of the graphs show exponential decay (the curve goes down as x increases)? Give a reason.

(2 marks)

$y = a \times b^x$

$b > 1$ means growth

$0 < b < 1$ means decay

A: $b = 1.2$ growth

B: $b = 0.8$ decay

C: $b = 4$ growth

D: $b = \frac{1}{3}$ decay

B and D

17 A histogram shows the heights of 71 trees. The table gives the frequency density for each class.

Height (h m)	Frequency density
$0 < h \leq 10$	1.2
$10 < h \leq 20$	2.5
$20 < h \leq 40$	1.5
$40 < h \leq 50$	0.4

Work out the frequency for each class and check that the total is 71. (2 marks)

frequency = fd x width

0–10: 1.2 × 10 = 12

10–20: 2.5 × 10 = 25

20–40: 1.5 × 20 = 30

40–50: 0.4 × 10 = 4

12 + 25 + 30 + 4 = 71

12, 25, 30, 4

18 Here are the first five terms of a quadratic sequence.

2 7 14 23 34

Find an expression, in terms of n, for the nth term of this sequence. (2 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, … have first differences 3, 5, 7, … and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences

5, 7, 9, 11. Second difference: 2, so the n^2 coefficient is 2 divided by 2 = 1

Subtract n^2 (1, 4, 9, 16, 25) from the sequence: 1, 3, 5, 7, 9, which has nth term $2n - 1$

So the nth term is $n^2 + 2n - 1$

Check

$n = 1$ gives $1 + 2 - 1 = 2$; $n = 2$ gives $4 + 4 - 1 = 7$; $n = 3$ gives $9 + 6 - 1 = 14$; $n = 4$ gives $16 + 8 - 1 = 23$; $n = 5$ gives $25 + 10 - 1 = 34$

$n^2 + 2n - 1$

19 Prove algebraically that the sum of an odd number and an even number is always odd. (2 marks)

Even number: $2n$

Odd number: $2m + 1$ (n, m integers)

$$2n + (2m + 1) = 2n + 2m + 1$$
$$= 2(n + m) + 1$$

$2(n + m)$ is a multiple of 2, so it is even; one more makes it odd

Sum is always odd

20 Simplify fully $\frac{2x^2 + 6x}{4x}$

(2 marks)

Factorise the top

$2x^2 + 6x = 2x(x + 3)$

$= \frac{2x(x + 3)}{4x}$

Cancel the common factor $2x$

$\boxed{\frac{x + 3}{2}}$