Weekly check: Grade 8

Worked solutions

GCSE Maths weekly check

All weekly checks online

1 Solve the simultaneous equations

$x^2 + y^2 = 20$

$y = 2x$

(4 marks)

Substitute $y = 2x$ into the circle equation: $x^2 + 4x^2 = 20$

So $5x^2 = 20$, giving $x^2 = 4$

$x = 2$ or $x = -2$

When $x = 2$, $y = 4$. When $x = -2$, $y = -4$

Check

$4 + 16 = 20$ for both pairs

$x = 2, y = 4$ and $x = -2, y = -4$

2 OAB is a triangle. OA = **a** and OB = **b**.

M is the midpoint of OA and N is the midpoint of OB.

Show that MN is parallel to AB.

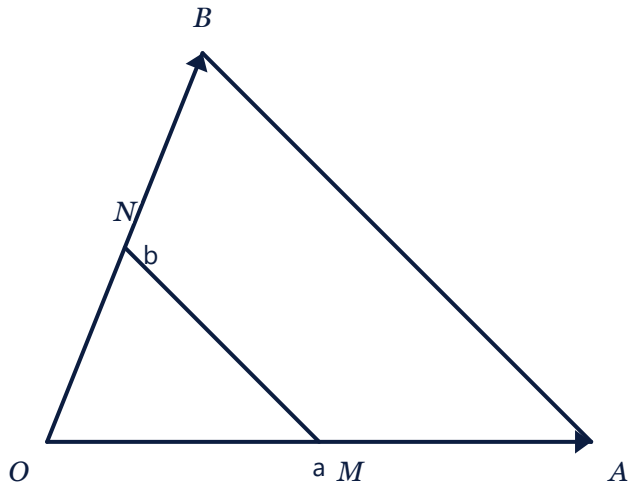


Diagram NOT accurately drawn

(3 marks)

$OM = \frac{1}{2}a$ and $ON = \frac{1}{2}b$

$MN = ON - OM = \frac{1}{2}b - \frac{1}{2}a = \frac{1}{2}(b - a)$

$AB = OB - OA = b - a$

$MN = \frac{1}{2}AB$, so MN is a scalar multiple of AB and is therefore parallel to AB

A scalar is an ordinary number multiplying a vector. Multiplying a non-zero vector by 1/2 halves its length without changing its direction, which establishes parallel lines

3 The curve **C** has equation $y = \cos x°$ for $0 \leq x \leq 360$.

Write down the coordinates of the minimum point of **C**.

(2 marks)

The cosine curve starts at (0, 1), falls to its lowest value of -1 at $x = 180$, and returns to 1 at $x = 360$

So the minimum point is (180, -1)

(180, -1)

4 OAB is a triangle.

Vector OA = **a** and vector OB = **b**.

M is the midpoint of OA and N is the midpoint of OB.

Prove that MN is parallel to AB.

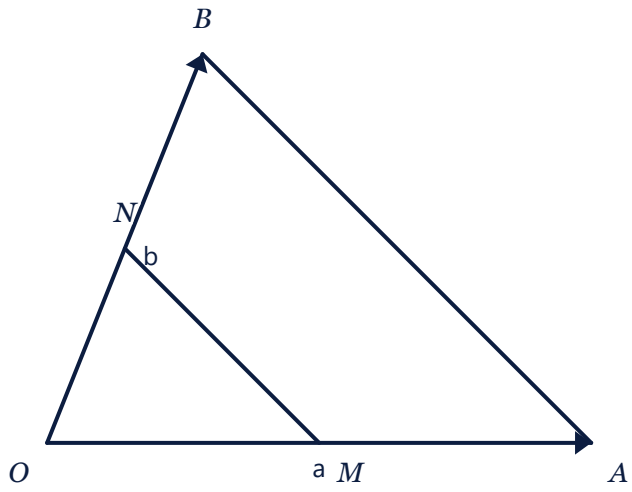


Diagram NOT accurately drawn

(4 marks)

Vector OM = $\frac{1}{2}$a because M is the midpoint of OA

Vector ON = $\frac{1}{2}$b because N is the midpoint of OB

Vector MN = ON - OM = $\frac{1}{2}$b - $\frac{1}{2}$a = $\frac{1}{2}$(b - a)

Vector AB = OB - OA = b - a

A scalar is an ordinary number. Multiplying a nonzero vector by a scalar changes its length (and may reverse it), but keeps its direction parallel

So MN = $\frac{1}{2}$AB, which means MN is a scalar multiple of AB

Therefore MN is parallel to AB

5 Expand and simplify (5 + 2√3)(5 − 2√3)

(2 marks)

difference of two squares

$(5 + 2\sqrt{3})(5 - 2\sqrt{3}) = 5^2 - (2\sqrt{3})^2$

$= 25 - 4 \times 3$

$= 25 - 12$

13

6 The graph of $y = \sin x$ for $0° \leq x \leq 360°$ has a maximum point at (90, 1).

a. Write down the coordinates of the maximum point of the graph of $y = \sin x - 3$

b. Write down the coordinates of the maximum point of the graph of $y = \sin (x - 30°)$

(2 marks)

a) $y = \sin x - 3$ translates the graph 3 units down, so (90, 1) maps to (90, -2)

b) $y = \sin(x - 30)$ translates the graph 30 degrees to the right, so (90, 1) maps to (120, 1)

7 Solve $(x - 3)(x + 7) \geq 0$

(2 marks)

The roots of $(x - 3)(x + 7) = 0$ are $x = 3$ and $x = -7$

The parabola is U-shaped, so the product is greater than or equal to zero outside the roots (including the roots themselves because the inequality is not strict)

So $x = 3$

$x = 3$

8 The number of members of a gym is modelled by the rule

$u_{n+1} = 1.2u_n - 50$

where u_n is the number of members after n years.

There are 500 members now, so $u_0 = 500$.

Work out the number of members the model predicts after 2 years.

(2 marks)

u0 is the starting value. The subscript n + 1 means the next year: put the current value un into the rule to get the next value. Repeat the rule twice for two years

$u1 = 1.2 \times 500 - 50 = 600 - 50 = 550$

$u2 = 1.2 \times 550 - 50 = 660 - 50 = 610$

610

9 A runner's velocity-time graph is a smooth curve. The table gives her velocity v m/s at time t seconds.

t: 0, 10, 20, 30 v: 0, 4.2, 6.6, 7.8

Use the trapezium rule with 3 strips of equal width to estimate the distance the runner travels in the first 30 seconds.

(3 marks)

Distance is the area under the velocity-time graph. Use 3 trapezia of width 10

First strip

$$\frac{1}{2} \times 10 \times (0 + 4.2) = 21$$

Second strip

$$\frac{1}{2} \times 10 \times (4.2 + 6.6) = 54$$

Third strip

$$\frac{1}{2} \times 10 \times (6.6 + 7.8) = 72$$

Estimate = 21 + 54 + 72 = 147 m

147 m

10 ABCDEFGH is a cuboid with base ABCD and top face EFGH, where E is directly above A, F is directly above B, G is directly above C and H is directly above D.

AB = 3 cm, BC = 4 cm and CG = 12 cm.

Work out the length of AG.

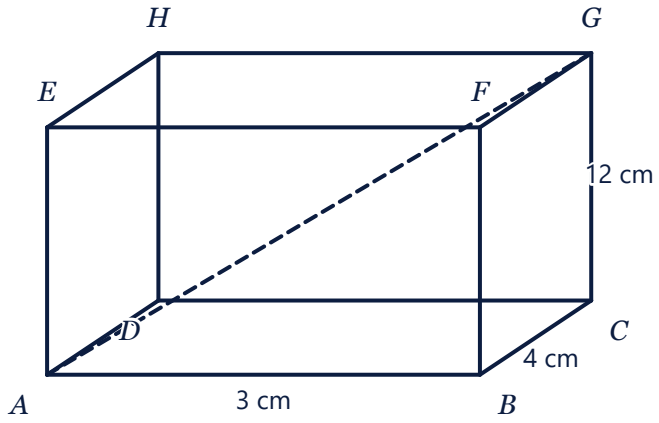


Diagram NOT accurately drawn

(2 marks)

First find the diagonal AC of the base using Pythagoras in triangle ABC: $AC^2 = 3^2 + 4^2 = 9 + 16 = 25$, so AC = 5 cm

AG is the space diagonal. Triangle ACG has a right angle at C because CG is vertical

$$AG^2 = AC^2 + CG^2 = 25 + 144 = 169$$

$$AG = \sqrt{169} = 13 \text{ cm}$$

Alternatively, in one step

$$AG = \sqrt{(3^2 + 4^2 + 12^2)} = \sqrt{169} = 13 \text{ cm}$$

13 cm

11 A circle has its centre at the origin O. The diameter of the circle is 16.

Find the equation of the circle.

(2 marks)

$$\text{radius} = 16 \div 2 = 8$$

$$r^2 = 8^2 = 64$$

$x^2 + y^2 = 64$

12 $f(x) = x^3 - 4x - 3$

Show that the equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$. (2 marks)

$f(2) = 8 - 8 - 3 = -3$

$f(3) = 27 - 12 - 3 = 12$

Sign change and f is continuous

A root lies between $x = 2$ and $x = 3$

13 Write $\frac{x+1}{2} + \frac{2x-1}{5}$ as a single fraction in its simplest form. (2 marks)

$$\frac{x+1}{2} + \frac{2x-1}{5} = \frac{5(x+1)}{10} + \frac{2(2x-1)}{10}$$

$$= \frac{5x + 5 + 4x - 2}{10}$$

$$\frac{9x + 3}{10}$$

14 The temperature, T °C, of a cup of tea is plotted against time, t minutes, and the points form a smooth curve going down.

At $t = 10$ a tangent is drawn to the curve. The tangent passes through the points (2, 80) and (14, 56).

a. Work out the gradient of the tangent.

b. What does this gradient tell you about the tea at $t = 10$? (3 marks)

a)

gradient $= \{56 - 80\}/\{14 - 2\}$

$= \{-24\}/\{12\}$

$= -2$

b)

gradient = rate of change of T

negative: temperature is falling

a) -2 b) cooling at 2 °C per minute

15 The curve **C** has equation $y = x^2 - 4x + 7$

The line **L** has equation $y = 2x - 2$

Show that **L** touches **C** at exactly one point, and find the coordinates of this point. (4 marks)

Set the expressions for y equal

$x^2 - 4x + 7 = 2x - 2$

Rearrange

$x^2 - 6x + 9 = 0$

Factorise

$(x - 3)^2 = 0$

Both factors in $(x - 3)(x - 3)$ are zero at the same value, $x = 3$; this is called a repeated root. Also, curve minus line $= (x - 3)^2$ is never negative, so the curve stays on one side of the line and touches it at this one point

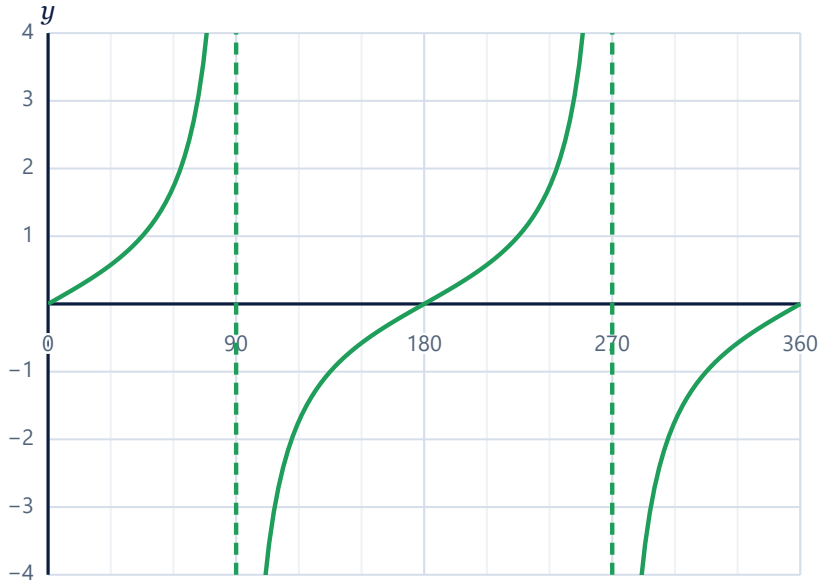
When $x = 3$, $y = 2(3) - 2 = 4$, so the point is $(3, 4)$

Check on the curve

$9 - 12 + 7 = 4$

16 Sketch the graph of $y = \tan x°$ for $0 \leq x \leq 360$.

Show clearly where the graph crosses the x-axis and mark the values of x where the graph has asymptotes.



(2 marks)

The tangent graph crosses the x-axis at $x = 0$, 180 and 360

It has vertical asymptotes at $x = 90$ and $x = 270$

Each branch increases from bottom left to top right, repeating every 180

A vertical asymptote is a line that the graph approaches while its values grow without bound. Here $\tan x = \sin x \div \cos x$ is undefined at 90° and 270° because $\cos x = 0$. Do not join the branches across those lines

17 The graph of $y = f(x)$ has a turning point at $(3, -5)$.

a. Write down the coordinates of the turning point of the graph of $y = f(x) + 4$

b. Write down the coordinates of the turning point of the graph of $y = f(x - 2)$

(2 marks)

a) $y = f(x) + 4$ translates the graph 4 units up, so $(3, -5)$ maps to $(3, -5 + 4) = (3, -1)$

b) $y = f(x - 2)$ translates the graph 2 units to the right, so $(3, -5)$ maps to $(3 + 2, -5) = (5, -5)$

For part (b), the original turning point occurs when the input of f is 3. In $f(x - 2)$, solve $x - 2 = 3$, giving $x = 5$. This explains why subtracting inside the function moves the point to the right

18 The circle C has equation $x^2 + y^2 = 61$

a. Show that the point A (5, −6) lies on C.

b. Does the point B (4, 7) lie on C? You must justify your answer.

(3 marks)

a) Substitute A into the left-hand side

$5^2 + (-6)^2 = 25 + 36 = 61$. This equals 61, so A lies on C

b) Substitute B

$4^2 + 7^2 = 16 + 49 = 65$. Since 65 is not equal to 61, B does not lie on C. In fact $65 > 61$, so B is outside the circle

19 Erin is asked to rationalise the denominator of $\frac{8}{\sqrt{6}}$

Here is her working.

$$\frac{8}{\sqrt{6}} = \frac{8 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{8\sqrt{6}}{36}$$

Erin has made one mistake.

Explain the mistake Erin has made.

(2 marks)

Erin has squared 6 instead of squaring $\sqrt{6}$

$\sqrt{6} \times \sqrt{6} = 6$, not 36

The correct answer is $\frac{8\sqrt{6}}{6}$, which simplifies to $\frac{4\sqrt{6}}{3}$

20 Rationalise the denominator of $\frac{12}{\sqrt{3}}$

Give your answer in its simplest form.

(2 marks)

Rationalise means remove the square root from the denominator. Multiplying top and bottom by $\sqrt{3}$ multiplies the fraction by $\sqrt{3}/\sqrt{3} = 1$, so it keeps the same value; $\sqrt{3} \times \sqrt{3} = 3$

Multiply the numerator and denominator by $\sqrt{3}$

$$\frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3}$$

Divide top and bottom by 3

$$\frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$\boxed{4\sqrt{3}}$