



- 1 Prove that $x^2 - 6x + 11$ is greater than 0 for all values of x .

(3 marks)

complete the square

$$\begin{aligned}x^2 - 6x + 11 &= (x - 3)^2 - 9 + 11 \\&= (x - 3)^2 + 2\end{aligned}$$

a squared number is never negative

$$\begin{aligned}&= (x - 3)^2 \geq 0 \\&= (x - 3)^2 + 2 \geq 2\end{aligned}$$

so it is always greater than 0

- 2 A bag contains n counters. 3 of the counters are red.

A counter is taken at random, replaced, and then a second counter is taken at random.

Write down an expression, in terms of n , for the probability that both counters are red.

(2 marks)

Each pick has $P(\text{red}) = \frac{3}{n}$ because the counter is replaced

$$P(\text{both red}) = \frac{3}{n} \times \frac{3}{n} = \frac{9}{n^2}$$

$$\frac{9}{n^2}$$

- 3 A regular hexagon is drawn inside a circle of radius r so that all six vertices lie on the circle. Each side of the hexagon is equal to r .

The perimeter of the hexagon is less than the circumference of the circle.

Use this to show that $\pi > 3$.

(3 marks)

$$\text{perimeter of hexagon} = 6 \times r = 6r$$

$$\text{circumference of circle} = 2\pi r$$

hexagon perimeter is less

$$= 6r < 2\pi r$$

divide both sides by $2r$ (positive)

$$= 3 < \pi$$

$\pi > 3$, as required

4 $y = -x^2 + 6x - 4$

By completing the square, find the coordinates of the turning point. State whether it is a maximum or a minimum.

(3 marks)

$$\begin{aligned} y &= -(x^2 - 6x) - 4 \\ &= -[(x - 3)^2 - 9] - 4 \\ &= -(x - 3)^2 + 9 - 4 \\ &= -(x - 3)^2 + 5 \end{aligned}$$

$(3, 5)$, a maximum

5 By completing the square, find the coordinates of the minimum point of the curve

$$y = x^2 + 10x - 2$$

(3 marks)

Complete the square

$$x^2 + 10x - 2 = (x + 5)^2 - 25 - 2 = (x + 5)^2 - 27$$

Check by expanding

$$x^2 + 10x + 25 - 27 = x^2 + 10x - 2$$

The minimum is when $x = -5$, so the minimum point is $(-5, -27)$

$(-5, -27)$

6 a. Write $x^2 - 6x + 11$ in the form $(x - a)^2 + b$, where a and b are integers.

b. Hence find the coordinates of the turning point of the curve $y = x^2 - 6x + 11$

(3 marks)

$$a) (x - 3)^2 = x^2 - 6x + 9, \text{ so } x^2 - 6x + 11 = (x - 3)^2 + 2$$

a) Check by expanding

$$x^2 - 6x + 9 + 2 = x^2 - 6x + 11$$

b) The minimum of $(x - 3)^2 + 2$ is when $x = 3$, giving the turning point $(3, 2)$

7 $f(x) = 2x + 1$ and $g(x) = x - 4$

Find $(fg)^{-1}(x)$.

(3 marks)

$$\begin{aligned}
 fg(x) &= 2(x - 4) + 1 = 2x - 7 \\
 y &= 2x - 7 \\
 +7 \quad y + 7 &= 2x \quad +7 \\
 x &= \frac{y + 7}{2} \\
 \boxed{(fg)^{-1}(x) = \frac{x + 7}{2}}
 \end{aligned}$$

8 Work out the exact value of $\sin 60^\circ \times \cos 30^\circ$.

Give your answer as a fraction.

(2 marks)

$$\begin{aligned}
 \sin 60^\circ &= \frac{\sqrt{3}}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2} \\
 \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} &= \frac{3}{4} \\
 \boxed{\frac{3}{4}}
 \end{aligned}$$

9 A bag contains $2x$ green counters and $(x + 4)$ yellow counters. A counter is taken at random.

The probability that it is green is $\frac{2}{5}$.

Work out the value of x .

(3 marks)

total counters = $2x + (x + 4) = 3x + 4$

$$\frac{2x}{(3x + 4)} = \frac{2}{5}$$

cross-multiply

$$\begin{aligned}
 10x &= 2(3x + 4) \\
 10x &= 6x + 8 \\
 -6x \quad 4x &= 8 \quad -6x \\
 \div 4 \quad x &= 2 \quad \div 4
 \end{aligned}$$

$x = 2$

- 10 A biased coin is flipped twice. The flips are independent.

The probability of getting heads on both flips is 0.64.

Work out the probability of getting heads on one flip of the coin.

(2 marks)

$$\text{Let } P(\text{heads}) = p. \text{ Then } p \times p = 0.64$$

$$p = \sqrt{0.64} = 0.8 \text{ (taking the positive root, since a probability cannot be negative)}$$

$$\boxed{0.8}$$

- 11 A box contains n counters. 5 of the counters are green.

A counter is taken at random and **not** replaced. A second counter is then taken at random.

Write down an expression, in terms of n , for the probability that both counters are green.

(2 marks)

$$P(\text{first green}) = \frac{5}{n}$$

After one green counter is removed, 4 green counters remain out of $n - 1$

$$P(\text{both green}) = \frac{5}{n} \times \frac{4}{n-1} = \frac{20}{n(n-1)}$$

$$\boxed{\frac{20}{n(n-1)}}$$

- 12 Work out the exact value of $(\sin 30^\circ + \cos 60^\circ) \times \tan 60^\circ$.

(2 marks)

$$\sin 30^\circ = \frac{1}{2} \text{ and } \cos 60^\circ = \frac{1}{2}, \text{ so } \sin 30^\circ + \cos 60^\circ = 1$$

$$\tan 60^\circ = \sqrt{3}$$

$$1 \times \sqrt{3} = \sqrt{3}$$

$$\boxed{\sqrt{3}}$$

- 13 Two shapes are mathematically similar. The smaller has area 12 cm^2 and the larger has area 27 cm^2 .

A side of the smaller shape is 4 cm long.

Work out the length of the matching side of the larger shape.

(2 marks)

$$\text{area scale factor} = \{27\} / \{12\} = \{9\} / \{4\}$$

$$\text{length scale factor} = \sqrt{\frac{9}{4}} = \{3\} / \{2\}$$

$$4 \times \{3\} / \{2\} = 6$$

$$\boxed{6 \text{ cm}}$$

- 14 Prove algebraically that the difference between the squares of two consecutive integers is equal to the sum of the two integers.

(3 marks)

Let the integers be n and $n + 1$

$$= (n + 1)^2 - n^2$$

$$= n^2 + 2n + 1 - n^2$$

$$= 2n + 1$$

$$= n + (n + 1)$$

= the sum of the two integers

- 15 Prove algebraically that the sum of the squares of any three consecutive integers is always 2 more than a multiple of 3.

(3 marks)

Let the integers be $n - 1, n, n + 1$

$$= (n - 1)^2 + n^2 + (n + 1)^2$$

$$= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$$

$$= 3n^2 + 2$$

= $3n^2 + 2$, 2 more than a multiple of 3

- 16 $y = 3x^2 - 12x + 5$

Write y in the form $a(x + b)^2 + c$ and write down the coordinates of the turning point.

(3 marks)

$$y = 3(x^2 - 4x) + 5$$

$$= 3[(x - 2)^2 - 4] + 5$$

$$= 3(x - 2)^2 - 12 + 5$$

$$= 3(x - 2)^2 - 7$$

Turning point (2, -7)

17

AB is a diameter of a circle with centre O.

C is a point on the circle, not at A or B.

Prove that angle $ACB = 90^\circ$.

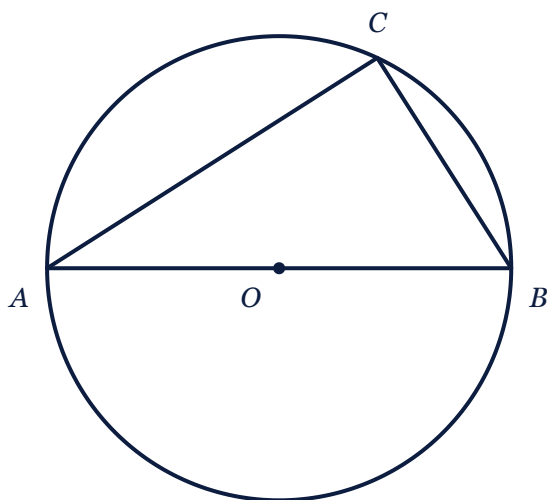


Diagram NOT accurately drawn

(3 marks)

Join O to C. $OA = OC$ and $OB = OC$ because all three are radii of the circle

Triangle OAC is isosceles, so angle $OAC = \text{angle } OCA$. Call this angle x

Triangle OBC is isosceles, so angle $OBC = \text{angle } OCB$. Call this angle y

In triangle ABC the three angles are angle $CAB = x$, angle $CBA = y$ and angle $ACB = x + y$

Angles in a triangle sum to 180° , so $x + y + (x + y) = 180^\circ$, which gives $2(x + y) = 180^\circ$

So $x + y = 90^\circ$, and therefore angle $ACB = 90^\circ$

This proves that the angle in a semicircle is a right angle

18 $y = x^2 + 6x + 4$

By completing the square, find the coordinates of the turning point and say whether it is a maximum or a minimum.

(3 marks)

$$\begin{aligned}x^2 + 6x + 4 &= (x + 3)^2 - 9 + 4 \\&= (x + 3)^2 - 5\end{aligned}$$

Turning point: $x + 3 = 0$

$(-3, -5)$, a minimum

19 Prove algebraically that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 12 for every integer n .

(3 marks)

Expand both squares

$$(3n + 1)^2 = 9n^2 + 6n + 1 \text{ and } (3n - 1)^2 = 9n^2 - 6n + 1$$

Subtract

$$(9n^2 + 6n + 1) - (9n^2 - 6n + 1) = 12n$$

$12n$ is 12 multiplied by an integer, so it is a multiple of 12

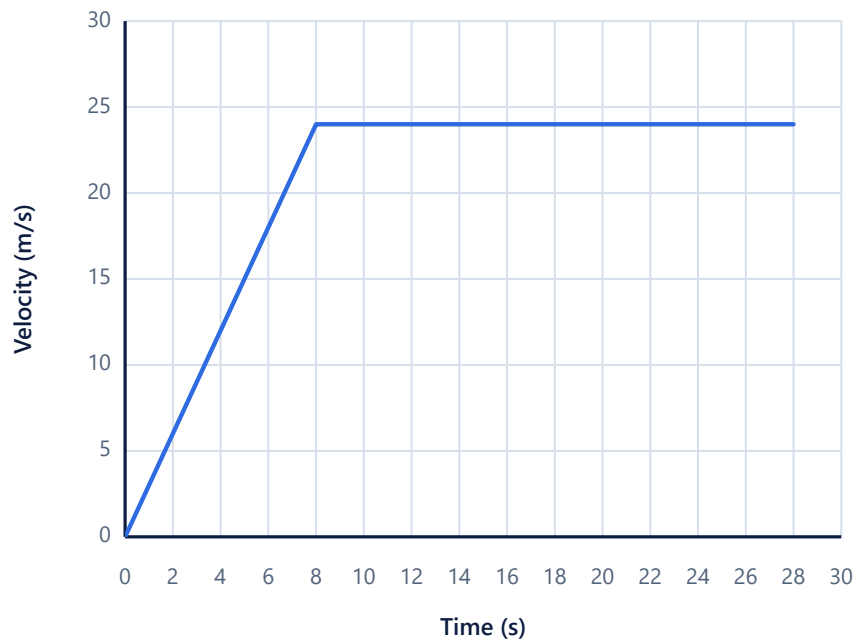
Therefore $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 12 for every integer n

20

A car starts from rest and accelerates uniformly to a velocity of 24 m/s in 8 seconds.

It then travels at a constant velocity of 24 m/s for a further 20 seconds.

- Work out the acceleration of the car during the first 8 seconds.
- Work out the total distance travelled by the car in the 28 seconds.



(3 marks)

a) Acceleration = change in velocity divided by time = $\frac{24}{8} = 3$

m/s²

b) Distance = area under the velocity-time graph

First 8 seconds (triangle)

$$\frac{1}{2} \times 8 \times 24 = 96 \text{ m}$$

Next 20 seconds (rectangle)

$$20 \times 24 = 480 \text{ m}$$

$$\text{Total distance} = 96 + 480 = 576 \text{ m}$$