



- 1 Write  $x^2 + 10x + 7$  in the form  $(x + a)^2 + b$ , where  $a$  and  $b$  are integers.

(2 marks)

$$= x^2 + 10x + 7$$

Half of 10 is 5

$$= (x + 5)^2 - 25 + 7$$

$$(x + 5)^2 - 18$$

- 2 The line with equation  $y = 3x + 5$  is parallel to the line with equation  $y = (2k - 1)x + 2$

Work out the value of  $k$ .

(2 marks)

parallel lines have equal gradients

$$\begin{array}{rcl} 2k - 1 & = & 3 \\ +1 & & \\ 2k & = & 4 \\ \div 2 & & \\ k & = & 2 \end{array}$$

$$k = 2$$

- 3 40 students took a typing test. Salma drew a cumulative frequency graph for their times.

Salma plotted each cumulative frequency against the midpoint of its class interval.

- Explain the mistake Salma made.
- Salma redraws the graph correctly. The correct graph shows a cumulative frequency of 34 at a time of 40 seconds. How many students took longer than 40 seconds?
- What percentage of the 40 students took longer than 40 seconds?

(3 marks)

A cumulative frequency tells you how many values are less than or equal to the top of the class, so it must be plotted at the upper class boundary

$$\text{Students taking longer than 40 seconds} = 40 - 34 = 6$$

$$6 \text{ out of } 40 = \frac{6}{40} = 15\%$$

- 4 Lena says, "The graph of  $y = 3 \times 2^x$  goes through the point (2, 36), because  $3 \times 2 = 6$  and  $6^2 = 36$ ."

Lena is wrong. Explain her mistake and work out the correct  $y$ -value when  $x = 2$ .

(2 marks)

Powers come before multiplying

Lena did  $(3 \times 2)^2 = 36$

It should be  $3 \times 2^2$

$$y = 3 \times 2^2$$

$$y = 3 \times 4$$

$$y = 12$$

She squared the 3 too.  $y = 12$

- 5 Dev buys a van for £15 000.

The value of the van depreciates by 12% each year.

Calculate the value of the van at the end of 3 years.

Give your answer to the nearest penny.

(3 marks)

The multiplier for a 12% decrease is 0.88

After 3 years the value is  $15000 \times 0.88^3$

$$0.88^3 = 0.681472, \text{ so the value is } 15000 \times 0.681472 = \\ \pounds 10222.08$$

£10222.08

- 6 A sector of a circle has radius 9 cm and angle  $120^\circ$ .

Work out the arc length of the sector.

Give your answer in terms of  $\pi$ .

(2 marks)

Fraction of the circle

$$\frac{120}{360} = \frac{1}{3}$$

$$\text{arc} = \frac{1}{3} \times 2 \times \pi \times 9$$

$$= \frac{1}{3} \times 18\pi$$

$6\pi$  cm

7

A straight line passes through the points A (2, 5) and B (6, 17).

Work out the gradient of the line.

(2 marks)

Gradient = change in y divided by change in x

Change in y =  $17 - 5 = 12$ . Change in x =  $6 - 2 = 4$

Gradient =  $12 \div 4 = 3$

3

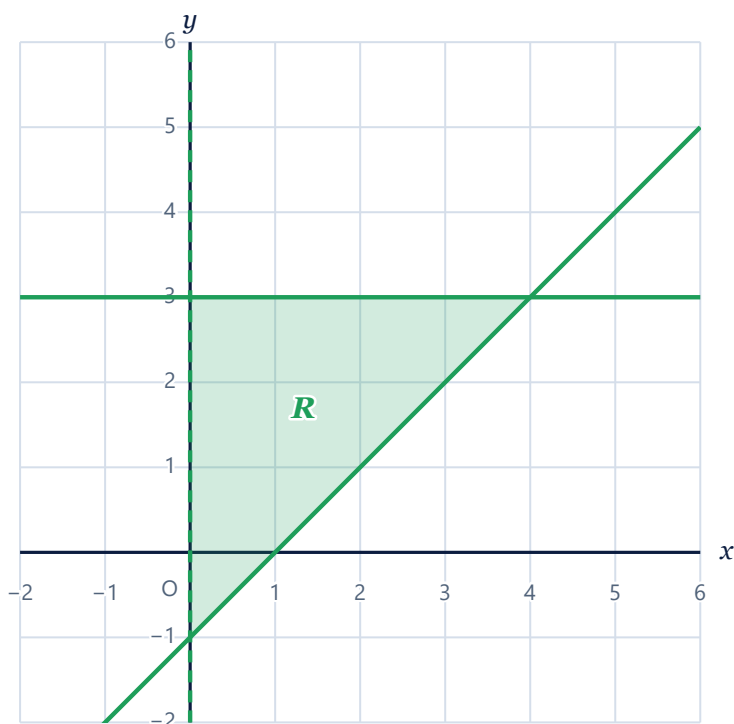
8 On the grid, shade the region that satisfies all three of these inequalities.

$$y \leq 3$$

$$y \geq x - 1$$

$$x > 0$$

Label the region R.



(3 marks)

Draw the horizontal line  $y = 3$  (solid) and keep the region below it

Draw the line  $y = x - 1$  (solid) through  $(0, -1)$  and  $(4, 3)$  and keep the region above it

Draw the line  $x = 0$ , the  $y$ -axis, dashed because  $x > 0$  is strict, and keep the region to the right

$R$  is the triangle with vertices  $(0, -1)$ ,  $(0, 3)$  and  $(4, 3)$

The stated corner coordinates are where the boundary lines meet. A dashed boundary is excluded from  $R$ , including any corner lying on that dashed line

9  $y$  is inversely proportional to  $x$ .

When  $x = 3$ ,  $y = 20$ .

Work out the value of  $y$  when  $x = 5$ .

(2 marks)

$$k = 3 \times 20 = 60, \text{ so } y = \frac{60}{x}$$

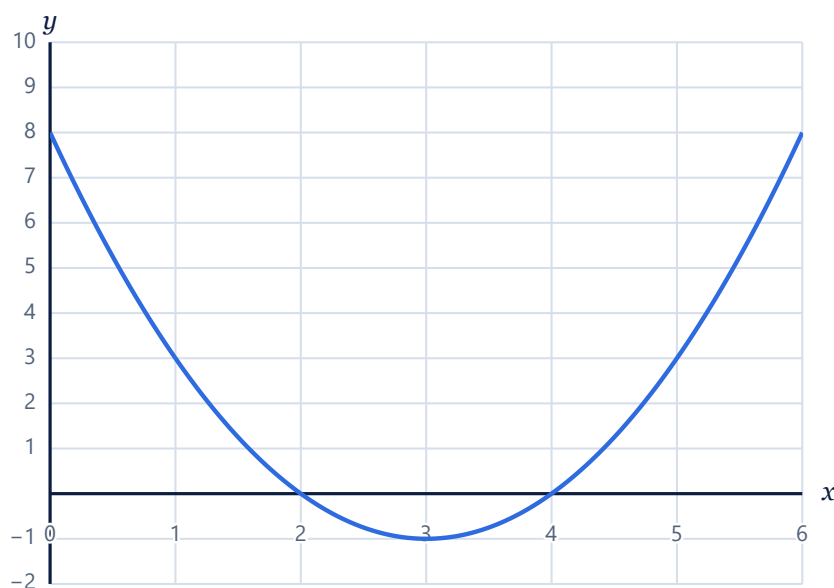
$$\text{When } x = 5, y = 60 \div 5 = 12$$

12

10 The curve  $y = x^2 - 6x + 8$  crosses the  $x$ -axis at  $x = 2$  and at  $x = 4$ .

a. Write down the solutions of  $x^2 - 6x + 8 = 0$

b. Write down the  $x$ -coordinate of the turning point of the curve.



(2 marks)

a) The solutions of  $x^2 - 6x + 8 = 0$  are the  $x$ -values where the curve crosses the  $x$ -axis:  $x = 2$  and  $x = 4$

b) By symmetry, the turning point is halfway between the roots:  $x = \frac{2 + 4}{2} = 3$

11 Without working out the squares, show that

$$53^2 - 47^2 = 600$$

(3 marks)

Use the difference of two squares

$$a^2 - b^2 = (a + b)(a - b)$$

$$\begin{aligned} 53^2 - 47^2 &= (53 + 47)(53 - 47) \\ &= 100 \times 6 = 600 \end{aligned}$$

12

Solve  $2x^2 + 5x - 12 = 0$

(3 marks)

Look for two numbers that multiply to  $2 \times (-12) = -24$  and add to 5: they are 8 and -3

Split the middle term

$$2x^2 + 8x - 3x - 12 = 2x(x + 4) - 3(x + 4) = (2x - 3)(x + 4)$$

Set each factor to zero

$$2x - 3 = 0 \text{ gives } x = \frac{3}{2}, \text{ and } x + 4 = 0 \text{ gives } x = -4$$

Check  $x = -4$ :  $2(16) - 20 - 12 = 32 - 32 = 0$ , correct

$$x = \frac{3}{2}, x = -4$$

13

The straight line L has equation  $y = -\frac{x}{3} + 5$

Find the equation of the straight line that is perpendicular to L and passes through the point (2, -1). (3 marks)

The gradient of L is  $-\frac{1}{3}$ , so the perpendicular gradient is 3,

$$\text{since } -\frac{1}{3} \times 3 = -1$$

The new line has the form  $y = 3x + c$

Substitute (2, -1)

$$-1 = 6 + c, \text{ so } c = -7$$

The equation is  $y = 3x - 7$

$$y = 3x - 7$$

14

A box contains 12 chocolates: 5 dark and 7 milk. Ravi takes two chocolates at random without replacement.

Work out the probability that at least one of the chocolates is dark.

(3 marks)

At least one dark =  $1 - P(\text{none dark})$

$$P(\text{both milk}) = \frac{7}{12} \times \frac{6}{11} = \frac{42}{132}$$

$$\frac{42}{132} = \frac{7}{22}$$

$$1 - \frac{7}{22} = \frac{15}{22}$$

$$\frac{15}{22}$$

15

A, B and C are points on a circle with centre O.

AOC is a diameter of the circle.

Angle CAB =  $37^\circ$ .

Work out the size of angle ACB. Give a reason for each stage of your working.

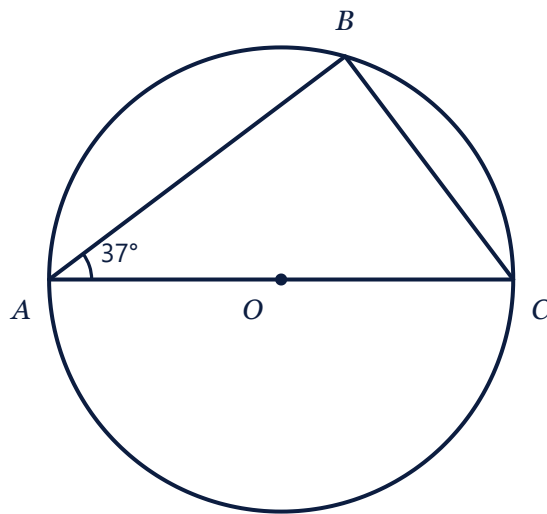


Diagram NOT accurately drawn

(3 marks)

Angle ABC =  $90^\circ$  because the angle in a semicircle is  $90^\circ$

Angles in a triangle add up to  $180^\circ$

$$\text{Angle ACB} = 180^\circ - 90^\circ - 37^\circ = 53^\circ$$

$$53^\circ$$

16 Prove that  $n^2 + n$  is even for every integer  $n$ .

(2 marks)

Factorise

$$n^2 + n = n(n + 1)$$

$n$  and  $n + 1$  are consecutive integers, so one of them must be even

The product of an even number with any integer is even

Therefore  $n^2 + n$  is even for every integer  $n$

17 a. Using the discriminant, explain why the equation  $x^2 + 3x + 7 = 0$  has no real solutions.

b. Write down the number of real solutions of the equation  $x^2 - 6x + 9 = 0$ . Give a reason for your answer.

(4 marks)

For  $ax^2 + bx + c = 0$ , the quadratic formula is  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . Match  $a$ ,  $b$  and  $c$  including their signs.  $\pm$  means calculate once with  $+$  and once with  $-$ , and divide the entire numerator by  $2a$

The discriminant of  $ax^2 + bx + c = 0$  is  $b^2 - 4ac$

For  $x^2 + 3x + 7 = 0$ : discriminant  $= 9 - 28 = -19$ . A negative number has no real square root, so the formula gives no real solutions

For  $x^2 - 6x + 9 = 0$ : discriminant  $= 36 - 36 = 0$ , so there is exactly one repeated real solution

In fact  $x^2 - 6x + 9 = (x - 3)^2$ , so the repeated solution is  $x = 3$

18 100 students were asked how they travel to school. The two-way table shows the results.

|         | Walk | Bus | Car | Total |
|---------|------|-----|-----|-------|
| Year 10 | 18   | 22  | 10  | 50    |
| Year 11 | 12   | 14  | 24  | 50    |
| Total   | 30   | 36  | 34  | 100   |

A student is chosen at random.

Given that the student travels by bus, work out the probability that they are in Year 11.

(2 marks)

Given bus: only the Bus column

$$\text{Total on the bus} = 36$$

$$\text{Year 11 on the bus} = 14$$

$$P = \frac{14}{36}$$

$$\boxed{\frac{7}{18}}$$

19

 $\mathbf{a} = (-2, 5)$  and  $\mathbf{b} = (3, -1)$ .Work out  $3\mathbf{a} - 2\mathbf{b}$  as a column vector.

(2 marks)

$$3\mathbf{a} = (-6, 15) \text{ and } 2\mathbf{b} = (6, -2)$$

$$3\mathbf{a} - 2\mathbf{b} = (-6 - 6, 15 - (-2)) = (-12, 17)$$

$$\boxed{(-12, 17)}$$

20

Triangle P has vertices at (2, 1), (4, 1) and (4, 2).

Triangle P is enlarged by scale factor  $-1$ , centre (0, 0), to give triangle Q.

Write down the coordinates of the vertices of triangle Q.

(2 marks)

Scale factor  $-1$  with centre (0, 0) sends each point  $(x, y)$  to  $(-x, -y)$

$$(2, 1) \rightarrow (-2, -1); (4, 1) \rightarrow (-4, -1); (4, 2) \rightarrow (-4, -2)$$

$$\boxed{(-2, -1), (-4, -1), (-4, -2)}$$