Spearman's rank correlation coefficient

Grade 7 | Worked solutions

GCSE Statistics topic practice

1 Seven football teams are ranked by their position in the league and by their average home attendance.

For these seven teams, $\Sigma d^2 = 14$, where d is the difference between the two ranks for a team.

a. Calculate Spearman's rank correlation coefficient. (2)

b. Interpret your answer to part (a) in context. (1)

(3 marks)

a)

$$r = 1 - \frac{6 \times \text{sum of } d^2}{n(n^2 - 1)}$$

$$r = 1 - \frac{6 \times 14}{7 \times (49 - 1)}$$

$$r = 1 - \frac{84}{336}$$

$$r = 1 - 0.25$$

$$r = 0.75$$

b)

0.75 shows fairly strong positive correlation

Teams higher in the league

tend to have higher attendances

a) 0.75 b) higher in league, higher attendance

2 Two judges, Judge X and Judge Y, each rank eight cakes at a village show. Rank 1 is the best cake.

Cake	A	B	C	D	E	F	G	H
Judge X	1	2	3	4	5	6	7	8
Judge Y	3	1	2	6	4	8	5	7

a. Calculate Spearman's rank correlation coefficient for these ranks. Give your answer correct to 3 decimal places. (3)

b. Interpret your answer to part (a) in context. (1)

(4 marks)

a) d = rank from X - rank from Y

d -2, 1, 1, -2, 1, -2, 2, 1

d^2 4, 1, 1, 4, 1, 4, 4, 1

sum of d^2 = 4 + 1 + 1 + 4 + 1 + 4 + 4 + 1

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sum of d^2 = 20

$$r = 1 - \frac{6 \times \text{sum of } d^2}{n(n^2 - 1)}$$

$$r = 1 - \frac{6 \times 20}{8 \times (64 - 1)}$$

$$r = 1 - \frac{120}{504}$$

$$r = 1 - 0.238...$$

r = 0.762

b)

0.762 is close to 1

Strong positive correlation

The two judges mostly agree on the order of the cakes

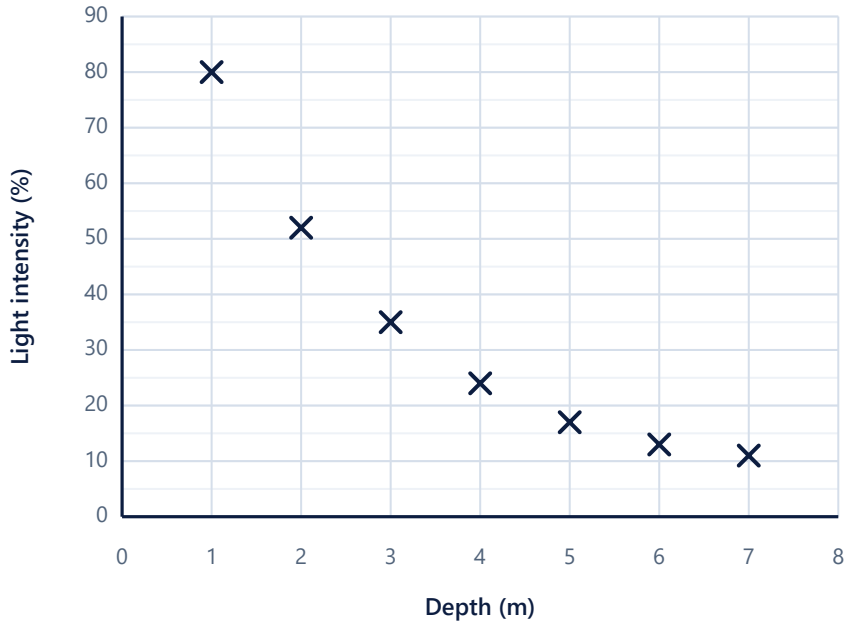
a) 0.762 b) strong positive, judges mostly agree

3 A diver measures the light intensity, as a percentage of the light at the surface, at seven depths in a lake. The scatter diagram shows her results.

a. Write down the value of Spearman's rank correlation coefficient for these data. Give a reason for your answer. (2)

b. Explain why Pearson's product moment correlation coefficient for these data is not −1. (1)

c. The diver takes one more reading. At a depth of 8 m the light intensity is 12%. Explain why Spearman's rank correlation coefficient for all eight readings is not −1. (1)



(4 marks)

a)

Coefficient = −1

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As the depth goes up,

the light intensity always goes down

so the two sets of ranks are exactly opposite

b)

The points lie on a curve, not on a straight line

c)

12 is more than the 11 at a depth of 7 m

Light intensity no longer always falls as depth rises

so the ranks are no longer exactly opposite

a) −1 b) not a straight line c) 12 > 11

4 The table shows the weekly screen time, in hours, and the mark in a test for each of six students.

Student	A	B	C	D	E	F
Screen time (hours)	12	5	20	8	15	3
Test mark	55	72	40	70	61	80
Screen time rank						
Test mark rank						

a. Complete the table by ranking the screen times and the test marks. Use rank 1 for the highest value. (2)

b. Calculate Spearman's rank correlation coefficient. Give your answer correct to 3 decimal places. (2)

c. Interpret your answer to part (b) in context. (1)

(5 marks)

a) Rank 1 = highest

Screen time ranks 3, 5, 1, 4, 2, 6

Test mark ranks 5, 2, 6, 3, 4, 1

b) d = screen time rank − test mark rank

d −2, 3, −5, 1, −2, 5

d^2 4, 9, 25, 1, 4, 25

$\text{sum of } d^2 = 4 + 9 + 25 + 1 + 4 + 25$

$\text{sum of } d^2 = 68$

$$r = 1 - \frac{6 \times \text{sum of } d^2}{n(n^2 - 1)}$$

$$r = 1 - \frac{6 \times 68}{6 \times (36 - 1)}$$

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$$r = 1 - \frac{408}{210}$$

$r = -0.943$

c)

-0.943 is close to -1

Strong negative correlation

Students with more screen time

tend to get lower test marks

a) ranks b) -0.943 c) strong negative

5 A manager and her deputy each rank the 10 applicants for a job. Spearman's rank correlation coefficient for their two sets of ranks is 0.6

a. Work out the value of Σd^2 for the two sets of ranks. (3)

b. The manager and the deputy both ranked applicant J in 10th place. Applicant J then withdraws, so the ranks of the other 9 applicants do not change. Calculate Spearman's rank correlation coefficient for the 9 remaining applicants. (2)

(5 marks)

a)

$$r = 1 - \frac{6 \times \text{sum of } d^2}{n(n^2 - 1)}$$

$$0.6 = 1 - \frac{6 \times \text{sum of } d^2}{10 \times (100 - 1)}$$

$$0.6 = 1 - \frac{6 \times \text{sum of } d^2}{990}$$

$$\frac{6 \times \text{sum of } d^2}{990} = 0.4$$

$$6 \times \text{sum of } d^2 = 396$$

$$\text{sum of } d^2 = 66$$

b) J had $d = 0$, so sum of d^2 is still 66, now $n = 9$

$$r = 1 - \frac{6 \times 66}{9 \times (81 - 1)}$$

$$r = 1 - \frac{396}{720}$$

$$r = 0.45$$

a) 66 b) 0.45

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