GCSE Statistics: Grade 8

Worked solutions

GCSE Statistics practice

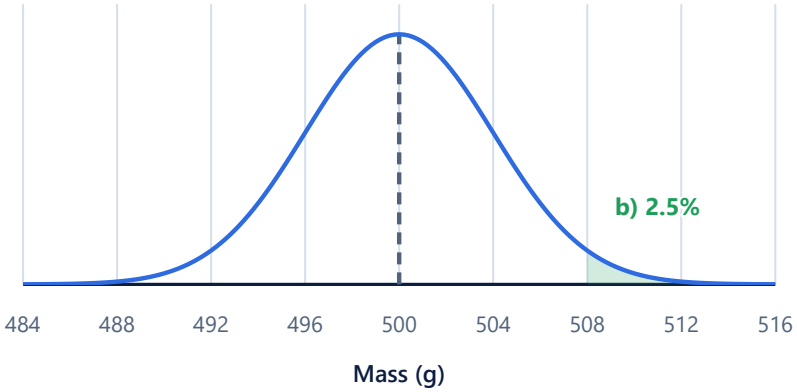
1 THE NORMAL DISTRIBUTION

A machine fills bags with flour. The masses of the bags are normally distributed with mean 500 g and standard deviation 4 g. The diagram shows this distribution.

a. Write down the percentage of bags with a mass between 496 g and 504 g.

b. Work out the percentage of bags with a mass of more than 508 g.

c. The machine fills 4000 bags in one day. Work out an estimate for the number of these bags with a mass of less than 488 g.



(5 marks)

a) 496 and 504 are 1 sd either side of the mean

68% of values are within 1 sd of the mean

b) 508 = 500 + 2 × 4, so it is 2 sd above the mean

95% of values are within 2 sd of the mean

100% - 95% = 5%

Half of this is above 508

5% ÷ 2 = 2.5%

c) 488 = 500 - 3 × 4, so it is 3 sd below the mean

99.7% of values are within 3 sd of the mean

100% - 99.7% = 0.3%

0.3% ÷ 2 = 0.15%

0.0015 × 4000 = 6

a) 68% b) 2.5% c) 6 bags

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2 **THE BINOMIAL DISTRIBUTION**

The probability that a sunflower seed germinates is 0.8. Kwame plants 3 of these seeds. Each seed germinates independently of the others.

You may use $(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$

a. Work out the probability that exactly 2 of the seeds germinate.

b. Work out the probability that at least 2 of the seeds germinate.

(4 marks)

a) $p = 0.8$ (germinates), $q = 1 - 0.8 = 0.2$ (does not)

Exactly 2 germinate is the term $3p^2q$

$$3p^2q = 3 \times 0.8^2 \times 0.2$$

$$3p^2q = 3 \times 0.64 \times 0.2$$

$$3p^2q = 0.384$$

b) At least 2 means exactly 2 or all 3

$$p^3 = 0.8^3$$

$$p^3 = 0.512$$

$$0.384 + 0.512 = 0.896$$

a) 0.384 b) 0.896

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3 **QUALITY ASSURANCE AND CONTROL CHARTS**

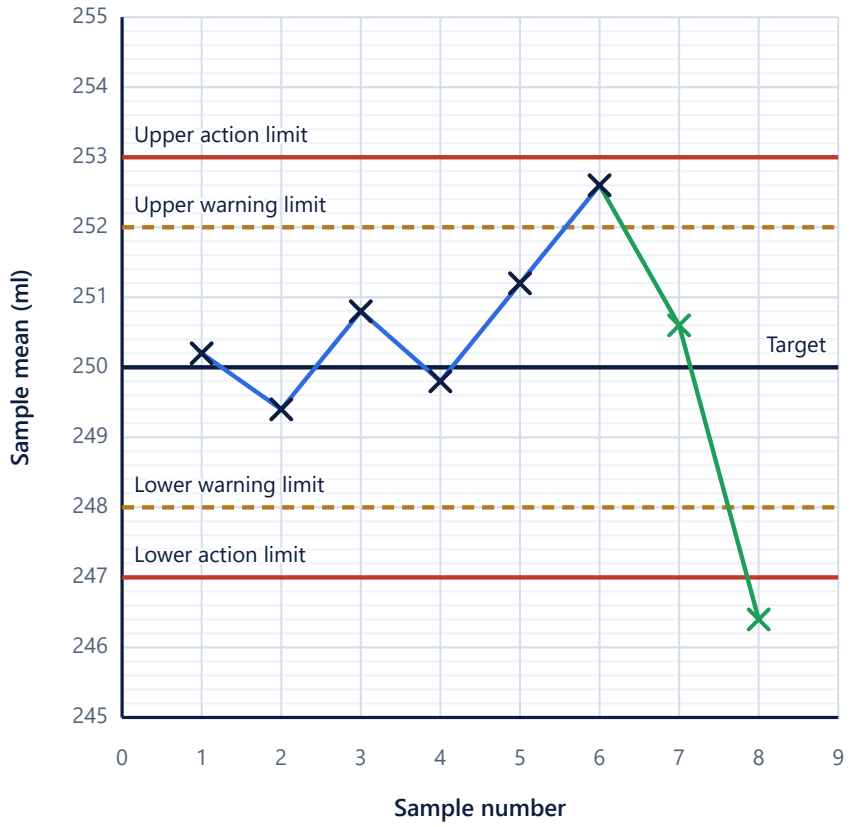
A machine fills bottles with juice. The target volume is 250 ml. Every hour a sample of 5 bottles is taken and the mean volume is plotted on the control chart. The chart shows the means of the first 6 samples.

The mean of sample 7 is 250.6 ml and the mean of sample 8 is 246.4 ml.

a. Plot the means of samples 7 and 8 on the control chart.

b. Describe what should be done after sample 6 is taken. Give a reason for your answer.

c. Describe what should be done after sample 8 is taken. Give a reason for your answer.



(5 marks)

a)

Plot (7, 250.6) and (8, 246.4)

b) Sample 6 has mean 252.6 ml

This is between the upper warning limit (252)

and the upper action limit (253)

Take another sample straight away

If that is also outside the warning limits, stop the machine

c) Sample 8 has mean 246.4 ml

This is below the lower action limit (247)

Stop the machine and reset it

a) plotted b) take another sample c) stop and reset

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4 **STANDARD DEVIATION FROM A FREQUENCY TABLE**

The table shows the number of goals scored by a hockey team in each of 20 matches.

Goals scored (x)	0	1	2	3	4
Frequency (f)	2	7	6	3	2

a. Show that the mean number of goals per match is 1.8

b. Calculate the standard deviation of the number of goals per match. Give your answer correct to 2 decimal places.

(4 marks)

a) Multiply each number of goals by its frequency

fx 0, 7, 12, 9, 8

sum of fx = 0 + 7 + 12 + 9 + 8

sum of fx = 36

mean = 36 ÷ 20

mean = 1.8

b) $fx^2 = f \times x \times x$

fx^2 0, 7, 24, 27, 32

sum of fx^2 = 0 + 7 + 24 + 27 + 32

sum of fx^2 = 90

$$sd = \sqrt{\left(\frac{\text{sum of } fx^2}{\text{sum of } f} - \text{mean}^2\right)}$$

$$sd = \sqrt{\left(\frac{90}{20} - 1.8^2\right)}$$

$$sd = \sqrt{(4.5 - 3.24)}$$

$$sd = \sqrt{1.26}$$

sd = 1.12 goals

a) 36 ÷ 20 = 1.8 b) 1.12

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5 **INTERPRETING PEARSON'S AND SPEARMAN'S COEFFICIENTS**

The scatter diagram shows the age, in years, and the value, in thousands of pounds, of 8 cars of the same model.

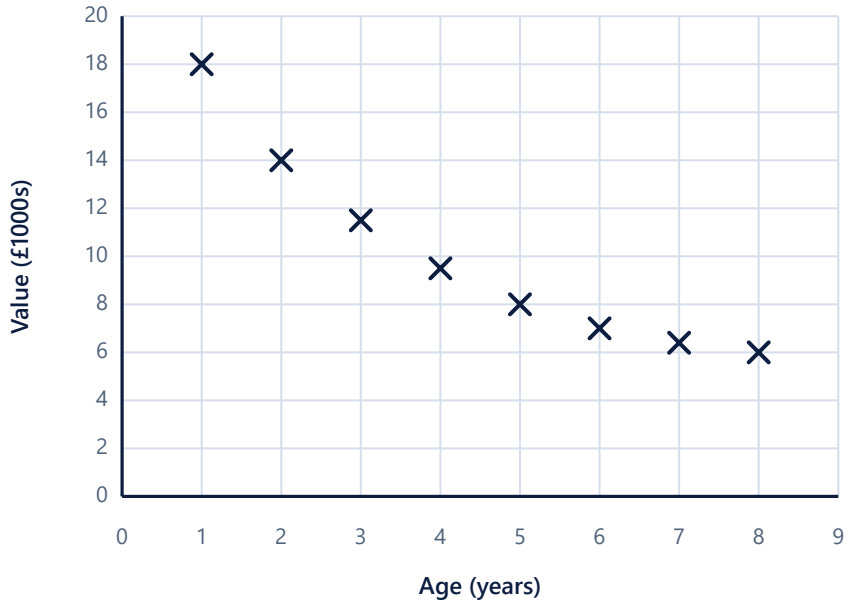
For these data, Spearman's rank correlation coefficient is −1 and Pearson's product moment correlation coefficient is −0.95

a. Explain what Spearman's rank correlation coefficient of −1 shows about these cars.

b. Interpret the value of Pearson's product moment correlation coefficient.

c. Use the scatter diagram to explain why Pearson's coefficient is not exactly −1

d. For the same 8 cars, Spearman's rank correlation coefficient between age and number of seats is 0.06. Interpret this value.



(4 marks)

a)

The ranks are in exactly opposite order

As age goes up, value always goes down

b) -0.95 is close to -1

Strong negative linear correlation

Older cars tend to have lower values

c)

Pearson measures how close the points are to a straight line

These points lie on a curve, not a straight line

d) 0.06 is very close to 0

Almost no correlation between the age of a car and its number of seats

a) opposite order b) strong negative c) curve d) none

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6 CALCULATING SKEW

The annual salaries of the staff at a company have mean £31 400, median £28 600 and standard deviation £6000.

a. Calculate the skew of the salaries.

b. Interpret your answer to part (a) in context.

c. Which average, the mean or the median, better represents a typical salary at this company? Give a reason for your answer.

(4 marks)

a)

$$\text{skew} = \frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$$

$$\text{skew} = \frac{3 \times (31400 - 28600)}{6000}$$

$$\text{skew} = \frac{3 \times 2800}{6000}$$

$$\text{skew} = \frac{8400}{6000}$$

$$\text{skew} = 1.4$$

b) The skew is positive

Most staff have lower salaries

A few staff have much higher salaries

c)

The median

It is not affected by the few very high salaries

a) 1.4 b) positive skew c) median

7 COMPARING DISTRIBUTIONS WITH MEAN AND STANDARD DEVIATION

A magazine tests the battery life, x hours, of 25 phones of Brand A. The results are summarised below.

$\Sigma x = 1500 \quad \Sigma x^2 = 91\,600$

a. Calculate the mean and the standard deviation of the battery life for Brand A.

b. For 25 phones of Brand B, the mean battery life is 57 hours and the standard deviation is 12.5 hours. Compare the battery life of the two brands.

(5 marks)

a)

$$\text{mean} = \frac{\text{sum of } x}{n}$$

$$\text{mean} = \frac{1500}{25}$$

$$\text{mean} = 60 \text{ hours}$$

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$$sd = \sqrt{\left(\frac{\text{sum of } x^2}{n} - \text{mean}^2\right)}$$

$$sd = \sqrt{\left(\frac{91600}{25} - 60^2\right)}$$

$$sd = \sqrt{(3664 - 3600)}$$

$$sd = \sqrt{64}$$

$$sd = 8 \text{ hours}$$

b) Compare the means

60 is greater than 57

On average, Brand A batteries last longer

Compare the standard deviations

8 is less than 12.5

Brand A battery life is more consistent

a) mean 60, sd 8 b) A longer on average and more consistent

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8 **MEAN SEASONAL VARIATION**

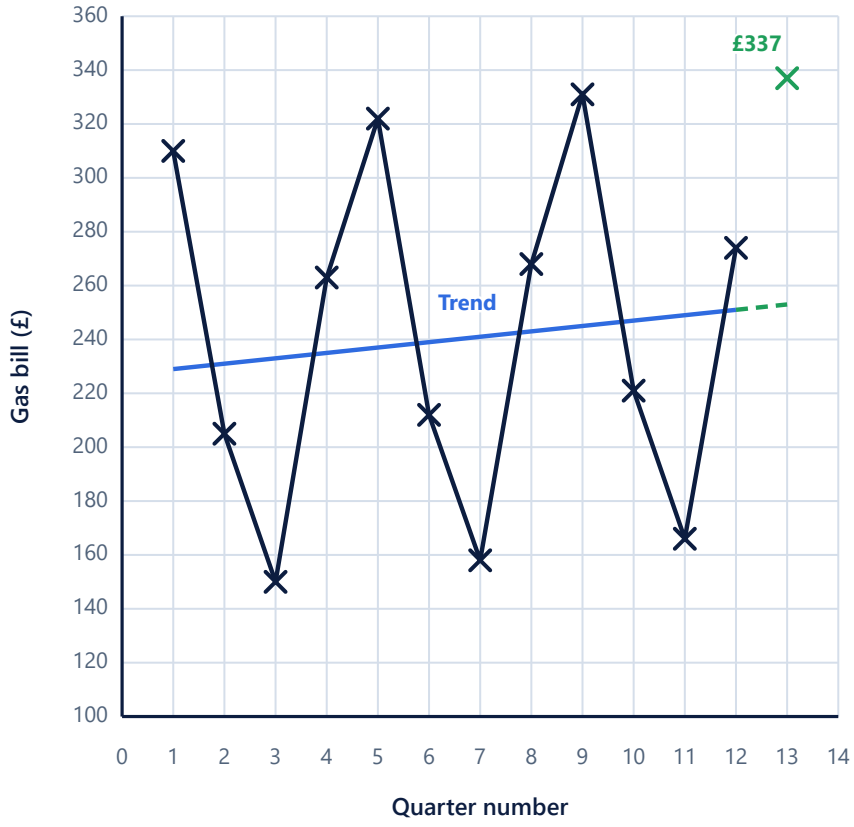
The time series graph shows the quarterly gas bills, in pounds, for a house over three years. Quarter 1 on the graph is January to March of Year 1. A trend line, labelled Trend, has been drawn.

The table shows the actual bill and the trend line value for the first quarter of each year.

Quarter number	Actual bill (£)	Trend value (£)
1 (Year 1)	310	229
5 (Year 2)	322	237
9 (Year 3)	331	245

a. Calculate the mean seasonal variation for the first quarter of the year.

b. The trend line goes up by £2 each quarter. Predict the gas bill for the first quarter of Year 4.



(5 marks)

a) seasonal variation = actual value − trend value

310 − 229 = 81

322 − 237 = 85

331 − 245 = 86

Find the mean of the three

81 + 85 + 86 = 252

252 ÷ 3 = 84

Mean seasonal variation = £84

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b) First quarter of Year 4 is quarter 13

4 quarters after quarter 9, so the trend goes up 4 × £2

trend value = 245 + 4 × 2

trend value = 253

prediction = trend value + mean seasonal variation

prediction = 253 + 84

prediction = £337

a) £84 b) £337

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