Standard deviation from a frequency table

Grade 8 | Worked solutions

GCSE Statistics topic practice

1 The table shows the number of siblings of each of 20 students. Some of the working for the mean and the standard deviation is shown.

Number of siblings (x)	0	1	2	3
Frequency (f)	4	9	5	2
fx	0	9		6
fx^2	0	9	20	

a. Complete the table. (1)

b. Work out the mean number of siblings. (1)

c. Calculate the standard deviation of the number of siblings. Give your answer correct to 2 decimal places. (2)

(4 marks)

a) $fx = f \times x$ and $fx^2 = f \times x \times x$

$5 \times 2 = 10$

$2 \times 3 \times 3 = 18$

The missing values are 10 and 18

b)

sum of $fx = 0 + 9 + 10 + 6$

sum of $fx = 25$

mean $= 25 \div 20$

mean $= 1.25$

The mean is 1.25 siblings

c)

sum of $fx^2 = 0 + 9 + 20 + 18$

sum of $fx^2 = 47$

$$sd = \sqrt{\left(\frac{\text{sum of } fx^2}{\text{sum of } f} - \text{mean}^2\right)}$$

$$sd = \sqrt{\left(\frac{47}{20} - 1.25^2\right)}$$

$$sd = \sqrt{(2.35 - 1.5625)}$$

$$sd = \sqrt{0.7875}$$

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sd = 0.89

a) 10 and 18 b) 1.25 c) 0.89

2 The table shows the number of goals scored by a hockey team in each of 20 matches.

Goals scored (x)	0	1	2	3	4
Frequency (f)	2	7	6	3	2

a. Show that the mean number of goals per match is 1.8 (2)

b. Calculate the standard deviation of the number of goals per match. Give your answer correct to 2 decimal places. (2)

(4 marks)

a) Multiply each number of goals by its frequency

fx 0, 7, 12, 9, 8

sum of fx = 0 + 7 + 12 + 9 + 8

sum of fx = 36

mean = 36 ÷ 20

mean = 1.8

b) $fx^2 = f \times x \times x$

fx^2 0, 7, 24, 27, 32

sum of fx^2 = 0 + 7 + 24 + 27 + 32

sum of fx^2 = 90

$$sd = \sqrt{\left(\frac{\text{sum of } fx^2}{\text{sum of } f} - \text{mean}^2\right)}$$

$$sd = \sqrt{\left(\frac{90}{20} - 1.8^2\right)}$$

$$sd = \sqrt{(4.5 - 3.24)}$$

$$sd = \sqrt{1.26}$$

sd = 1.12 goals

a) 36 ÷ 20 = 1.8 b) 1.12

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3 A machine packs sweets into bags. Asif counts the number of sweets, x, in each of 50 bags and puts his results in a frequency table. He works out these totals.

$\Sigma f = 50 \quad \Sigma fx = 1200 \quad \Sigma fx^2 = 29\,250$

a. Calculate the mean and the standard deviation of the number of sweets in a bag. (3)

b. For bags packed by a second machine, the standard deviation of the number of sweets is 1.8. Which machine is more consistent? Give a reason for your answer. (1)

(4 marks)

a)

$$\text{mean} = \frac{\text{sum of } fx}{\text{sum of } f}$$

$$\text{mean} = \frac{1200}{50}$$

$$\text{mean} = 24 \text{ sweets}$$

$$\text{sd} = \sqrt{\left(\frac{\text{sum of } fx^2}{\text{sum of } f} - \text{mean}^2\right)}$$

$$\text{sd} = \sqrt{\left(\frac{29250}{50} - 24^2\right)}$$

$$\text{sd} = \sqrt{(585 - 576)}$$

$$\text{sd} = \sqrt{9}$$

$$\text{sd} = 3 \text{ sweets}$$

b) 1.8 is less than 3

The second machine is more consistent

It has the lower standard deviation

a) mean 24, sd 3 b) second machine, lower sd

4 The table shows the times, t minutes, that 20 customers waited for a table at a restaurant.

Time (t minutes)	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	8
$20 < t \leq 30$	6
$30 < t \leq 40$	3

a. Calculate an estimate for the mean waiting time. (2)

b. Calculate an estimate for the standard deviation of the waiting times. Give your answer correct to 1 decimal place. (2)

c. Explain why your answers are only estimates. (1)

(5 marks)

a) Use the midpoints 5, 15, 25, 35 for x

fx 15, 120, 150, 105

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sum of fx = 15 + 120 + 150 + 105

sum of fx = 390

mean = 390 ÷ 20

mean = 19.5 minutes

b) fx^2 = f x midpoint x midpoint

fx^2 75, 1800, 3750, 3675

sum of fx^2 = 75 + 1800 + 3750 + 3675

sum of fx^2 = 9300

$$sd = \sqrt{\left(\frac{\text{sum of } fx^2}{\text{sum of } f} - \text{mean}^2\right)}$$

$$sd = \sqrt{\left(\frac{9300}{20} - 19.5^2\right)}$$

$$sd = \sqrt{(465 - 380.25)}$$

$$sd = \sqrt{84.75}$$

sd = 9.2 minutes

c)

The exact waiting times are not known

The midpoints are used in place of the real times

a) 19.5 minutes b) 9.2 minutes c) exact times not known

5 The table shows information about the number of bedrooms in each house on a street.

Number of bedrooms (x)	1	2	3	4
Frequency (f)	4	6	k	4

The mean number of bedrooms is 2.5

a. Show that $k = 6$ (2)

b. Calculate the standard deviation of the number of bedrooms. Give your answer correct to 2 decimal places. (3)

(5 marks)

a)

sum of fx = 4 + 12 + 3k + 16

sum of fx = 32 + 3k

sum of f = 14 + k

mean = sum of fx ÷ sum of f

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$$\frac{32 + 3k}{14 + k} = 2.5$$

$$32 + 3k = 2.5(14 + k)$$

$$32 + 3k = 35 + 2.5k$$

$$0.5k = 3$$

$$k = 6$$

So there are 6 houses with 3 bedrooms

b) sum of $f = 14 + 6 = 20$

$$\text{sum of } fx^2 = 4 \times 1 + 6 \times 4 + 6 \times 9 + 4 \times 16$$

$$\text{sum of } fx^2 = 4 + 24 + 54 + 64$$

$$\text{sum of } fx^2 = 146$$

$$\text{sd} = \sqrt{\left(\frac{\text{sum of } fx^2}{\text{sum of } f} - \text{mean}^2\right)}$$

$$\text{sd} = \sqrt{\left(\frac{146}{20} - 2.5^2\right)}$$

$$\text{sd} = \sqrt{(7.3 - 6.25)}$$

$$\text{sd} = \sqrt{1.05}$$

$$\text{sd} = 1.02$$

a) $k = 6$ shown b) 1.02

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