The binomial distribution

Grade 8 | Worked solutions

GCSE Statistics topic practice

1 The probability that Mia's train to work is late on any day is 0.3. Mia takes the train on 2 days. Whether the train is late on one day is independent of whether it is late on the other day.

You may use $(p + q)^2 = p^2 + 2pq + q^2$

a. Work out the probability that the train is late on both days. (1)

b. Work out the probability that the train is late on exactly one of the two days. (2)

(3 marks)

a) $p = 0.3$ (late), $q = 1 - 0.3 = 0.7$ (not late)

Late on both days is the term p^2

$p^2 = 0.3^2$

$p^2 = 0.09$

P(Late on both days) = 0.09

b) Late on exactly one day is the term $2pq$

$2pq = 2 \times 0.3 \times 0.7$

$2pq = 0.42$

a) 0.09 b) 0.42

2 The probability that a sunflower seed germinates is 0.8. Kwame plants 3 of these seeds. Each seed germinates independently of the others.

You may use $(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$

a. Work out the probability that exactly 2 of the seeds germinate. (2)

b. Work out the probability that at least 2 of the seeds germinate. (2)

(4 marks)

a) $p = 0.8$ (germinates), $q = 1 - 0.8 = 0.2$ (does not)

Exactly 2 germinate is the term $3p^2q$

$3p^2q = 3 \times 0.8^2 \times 0.2$

$3p^2q = 3 \times 0.64 \times 0.2$

$3p^2q = 0.384$

P(exactly 2 germinate) = 0.384

b) At least 2 means exactly 2 or all 3

$p^3 = 0.8^3$

$p^3 = 0.512$

$0.384 + 0.512 = 0.896$

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a) 0.384 b) 0.896

3 A quiz has 4 multiple choice questions. Each question has 4 possible answers and only one of them is correct. Sunil guesses the answer to every question.

You may use $(p + q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

a. Give two reasons why a binomial distribution is a suitable model for the number of questions Sunil gets correct. (2)

b. Work out the probability that Sunil gets exactly 1 question correct. Give your answer correct to 3 significant figures. (2)

(4 marks)

a) Any two of these reasons

There is a fixed number of trials (4 questions)

Each trial has two outcomes, correct or not correct

The probability of correct is the same each time (0.25)

The guesses are independent of each other

b) $p = 1 \div 4 = 0.25$ (correct), $q = 0.75$ (not correct)

Exactly 1 correct is the term $4pq^3$

$4pq^3 = 4 \times 0.25 \times 0.75^3$

$4pq^3 = 1 \times 0.421875$

$4pq^3 = 0.422$ (3 significant figures)

a) two reasons b) 0.422

4 A factory makes phone chargers. The probability that a charger is faulty is 0.1. The chargers are packed in boxes of 5. Each charger is faulty independently of the others.

You may use $(p + q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

a. Work out the probability that a box contains at most 1 faulty charger. Give your answer correct to 3 significant figures. (3)

b. An inspector checks 200 boxes. Work out an estimate for the number of these boxes that contain more than 1 faulty charger. (2)

(5 marks)

a) $p = 0.1$ (faulty), $q = 1 - 0.1 = 0.9$ (not faulty)

At most 1 faulty means 0 faulty or 1 faulty

$q^5 = 0.9^5$

$q^5 = 0.59049$

$5pq^4 = 5 \times 0.1 \times 0.9^4$

$5pq^4 = 0.32805$

$0.59049 + 0.32805 = 0.91854$

P(at most 1 faulty) = 0.919 (3 significant figures)

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b) More than 1 faulty is every other outcome

$1 - 0.91854 = 0.08146$

$0.08146 \times 200 = 16.292$

About 16 boxes

a) 0.919 b) 16 boxes

5 Zara takes 3 penalty shots. The probability that she scores is the same for each shot, and each shot is independent of the others. The probability that Zara scores with all 3 shots is 0.216

You may use $(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$

a. Show that the probability that Zara scores with any one shot is 0.6 (1)

b. Work out the probability that Zara scores with exactly 1 of the 3 shots. (2)

c. Work out the probability that Zara scores with at least 1 of the 3 shots. (2)

(5 marks)

a) Scoring with all 3 shots is the term p^3, so $p^3 = 0.216$

$0.6^3 = 0.6 \times 0.6 \times 0.6$

$0.6^3 = 0.216$

So $p = 0.6$

b) $q = 1 - 0.6 = 0.4$ (does not score)

Exactly 1 goal is the term $3pq^2$

$3pq^2 = 3 \times 0.6 \times 0.4^2$

$3pq^2 = 3 \times 0.6 \times 0.16$

$3pq^2 = 0.288$

P(exactly 1 goal) = 0.288

c) At least 1 goal is every outcome except no goals

$q^3 = 0.4^3$

$q^3 = 0.064$

$1 - 0.064 = 0.936$

a) $0.6^3 = 0.216$ b) 0.288 c) 0.936

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