# Binomial probability problems

Grade 9 | Worked solutions

**GCSE Statistics topic practice**

**1** Farah catches a train to work on 3 mornings. She models the number of mornings on which her train is late with a binomial distribution. The probability that the train is late on any morning is 0.2

You may use $(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$

a. Work out the probability that the train is late on exactly one of the 3 mornings. (2)

b. Write down one assumption Farah has made by using a binomial distribution. (1)

(3 marks)

a) $p = 0.2$ (late), $q = 1 - 0.2 = 0.8$ (not late)

Exactly one late is the term $3pq^2$

$3pq^2 = 3 \times 0.2 \times 0.8^2$

$3pq^2 = 3 \times 0.2 \times 0.64$

$3pq^2 = 0.384$

b) One of these

Being late on one morning is independent of the others

The probability of being late is the same every morning

**a) 0.384 b) mornings are independent**

**2** A quiz has 4 multiple choice questions. Each question has 3 possible answers and only one of them is correct. Tomasz guesses the answer to every question. The table shows the probability distribution of the number of questions he gets correct.

| Number correct | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Probability | $\frac{16}{81}$ | $\frac{32}{81}$ | | $\frac{8}{81}$ | |

You may use $(p + q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

a. Complete the table. (2)

b. To pass the quiz Tomasz needs at least 3 correct answers. Work out the probability that he passes. (1)

c. 162 students each guess the answer to every question. Work out an estimate for the number of these students who pass. (1)

(4 marks)

a) $p = \frac{1}{3}$ (correct), $q = \frac{2}{3}$ (wrong)

2 correct is the term $6p^2q^2$

$6p^2q^2 = 6 \times \frac{1}{9} \times \frac{4}{9}$

$6p^2q^2 = \frac{24}{81}$


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4 correct is the term $p^4$

$$p^4 = \frac{1}{81}$$

Check that the top numbers add up to 81

$$16 + 32 + 24 + 8 + 1 = 81$$

b) At least 3 means 3 or 4 correct

$$P(\text{pass}) = \frac{8}{81} + \frac{1}{81}$$

$$P(\text{pass}) = \frac{9}{81}$$

$$P(\text{pass}) = \frac{1}{9}$$

c) Expected number = probability x number of students

$$\frac{1}{9} \times 162 = 18$$

a) $\frac{24}{81}$ and $\frac{1}{81}$ b) $\frac{1}{9}$ c) 18

**3** A factory makes light bulbs and sells them in packs of 4. Each bulb works with probability $p$, independently of the other bulbs. The probability that all 4 bulbs in a pack work is 0.6561

You may use $(p + q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

a. Show that $p = 0.9$ (1)

b. Work out the probability that a pack has at most one faulty bulb. (2)

c. A shop buys 500 packs. Work out an estimate for the number of these packs with two or more faulty bulbs. (1)

(4 marks)

a) All 4 work is the term $p^4$

$$p^4 = 0.6561$$

$$p = \sqrt{(\sqrt{0.6561})}$$

$$p = \sqrt{0.81}$$

$$p = 0.9$$

b) $q = 1 - 0.9 = 0.1$ (faulty)

At most one faulty means none faulty or exactly one faulty

$$p^4 = 0.6561$$

$$4p^3q = 4 \times 0.9^3 \times 0.1$$

$$4p^3q = 4 \times 0.729 \times 0.1$$

$$4p^3q = 0.2916$$


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$0.6561 + 0.2916 = 0.9477$

c) Two or more faulty is everything else

$1 - 0.9477 = 0.0523$

$0.0523 \times 500 = 26.15$

About 26 packs

**a) $0.9^4 = 0.6561$ b) 0.9477 c) 26 packs**

**4** A garden centre sells sunflower seeds in trays of 5. Each seed germinates with probability 0.8. The number of seeds that germinate in a tray is modelled with a binomial distribution.

You may use $(p + q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

a. Work out the probability that exactly 4 of the seeds in a tray germinate. (2)

b. Work out the probability that at least 4 of the seeds in a tray germinate. Give your answer correct to 3 decimal places. (2)

c. The 5 seeds in a tray share the same soil and are watered together. Explain why a binomial distribution may not be a suitable model. (1)

(5 marks)

a) $p = 0.8$ (germinates), $q = 0.2$ (does not)

Exactly 4 germinate is the term $5p^4q$

$5p^4q = 5 \times 0.8^4 \times 0.2$

$5p^4q = 5 \times 0.4096 \times 0.2$

$5p^4q = 0.4096$

b) At least 4 means exactly 4 or all 5

$p^5 = 0.8^5$

$p^5 = 0.32768$

$0.4096 + 0.32768 = 0.73728$

0.737 to 3 decimal places

c)

The seeds may not be independent of each other

Poor soil or watering would affect every seed in the tray

**a) 0.4096 b) 0.737 c) seeds not independent**


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**5** Mei shoots 3 arrows at a target. Each arrow hits the target with probability $p$, independently of the other arrows. The probability that at least one of the 3 arrows hits the target is 0.973

You may use $(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$

a. Show that $p = 0.7$ (2)

b. Work out the probability that exactly 2 of the 3 arrows hit the target. (2)

c. Mei says, "The most likely result is that all 3 arrows hit the target." Is Mei right? Give a reason for your answer. (1)

(5 marks)

a) At least one hit = 1 - P(all 3 miss)

All 3 miss is the term $q^3$

$q^3 = 1 - 0.973$

$q^3 = 0.027$

$0.3 \times 0.3 \times 0.3 = 0.027$

$q = 0.3$

$p = 1 - 0.3$

$p = 0.7$

b) Exactly 2 hit is the term $3p^2q$

$3p^2q = 3 \times 0.7^2 \times 0.3$

$3p^2q = 3 \times 0.49 \times 0.3$

$3p^2q = 0.441$

c) All 3 hit is the term $p^3$

$p^3 = 0.7^3$

$p^3 = 0.343$

0.343 is less than 0.441

Mei is not right, exactly 2 hits is more likely

a) $q^3 = 0.027$, $q = 0.3$, $p = 0.7$ b) 0.441 c) No


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