Conditional probability without replacement

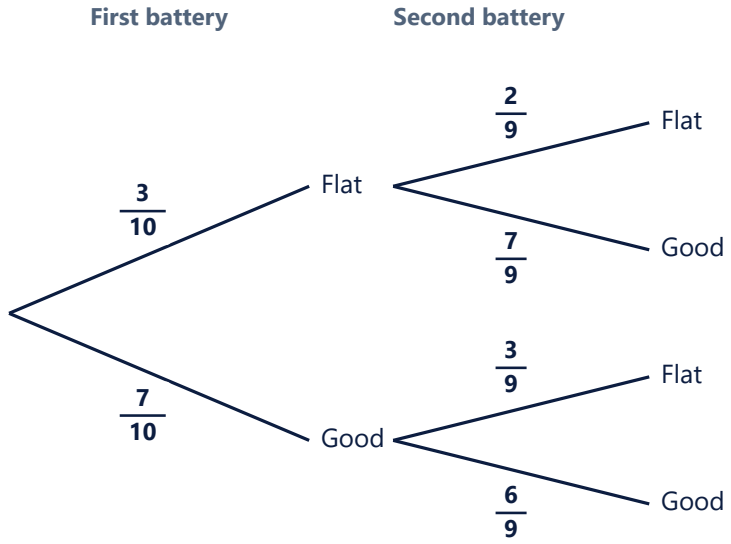
Grade 9 | Worked solutions

GCSE Statistics topic practice

1 A box contains 10 batteries. 3 of the batteries are flat and 7 are good. Aisha takes two batteries from the box at random, without replacement. The probability tree diagram shows this information.

a. Work out the probability that both batteries are flat. (1)

b. Work out the probability that exactly one of the two batteries is flat. (2)



(3 marks)

a) Multiply along the branches

$$P(\text{flat, flat}) = \frac{3}{10} \times \frac{2}{9}$$

$$P(\text{flat, flat}) = \frac{6}{90}$$

$$P(\text{flat, flat}) = \frac{1}{15}$$

b) There are two ways to get exactly one flat battery

$$P(\text{flat, good}) = \frac{3}{10} \times \frac{7}{9}$$

$$P(\text{flat, good}) = \frac{21}{90}$$

$$P(\text{good, flat}) = \frac{7}{10} \times \frac{3}{9}$$

$$P(\text{good, flat}) = \frac{21}{90}$$

Add the two ways

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$$\frac{21}{90} + \frac{21}{90} = \frac{42}{90}$$

$$\frac{42}{90} = \frac{7}{15}$$

a) $\frac{1}{15}$ b) $\frac{7}{15}$

2 A bag contains 5 mints and 3 toffees. Callum takes three sweets from the bag at random, one after the other, and eats them.

a. Work out the probability that the three sweets are all the same type. (3)

b. Work out the probability that Callum eats at least one sweet of each type. (1)

(4 marks)

a) The sweets are not replaced

Three mints

$$P(M, M, M) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$$

$$P(M, M, M) = \frac{60}{336}$$

Three toffees

$$P(T, T, T) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$$

$$P(T, T, T) = \frac{6}{336}$$

Add the two ways

$$\frac{60}{336} + \frac{6}{336} = \frac{66}{336}$$

$$\frac{66}{336} = \frac{11}{56}$$

b) At least one of each means not all the same

$$1 - \frac{11}{56} = \frac{45}{56}$$

a) $\frac{11}{56}$ b) $\frac{45}{56}$

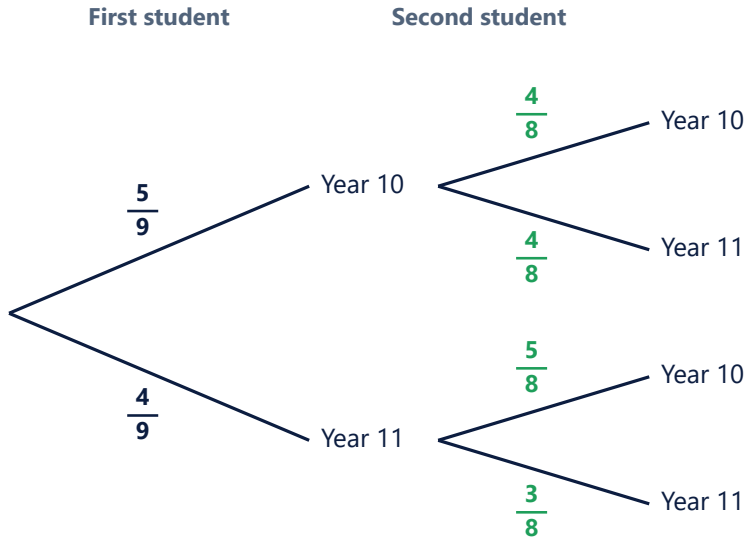
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3 A focus group is made up of 5 students from Year 10 and 4 students from Year 11. A researcher picks two of these 9 students at random, one after the other, to interview. No student can be picked twice.

a. Complete the probability tree diagram. (1)

b. Work out the probability that the two students picked are from the same year group. (2)

c. Given that the two students picked are from the same year group, work out the probability that they are both from Year 11. (2)



(5 marks)

a) After the first pick there are 8 students left

After Year 10 there are 4 Year 10 and 4 Year 11 Left

$\frac{4}{8}$ and $\frac{4}{8}$

After Year 11 there are 5 Year 10 and 3 Year 11 Left

$\frac{5}{8}$ and $\frac{3}{8}$

b) Multiply along the branches

$$P(\text{both Year 10}) = \frac{5}{9} \times \frac{4}{8}$$

$$P(\text{both Year 10}) = \frac{20}{72}$$

$$P(\text{both Year 11}) = \frac{4}{9} \times \frac{3}{8}$$

$$P(\text{both Year 11}) = \frac{12}{72}$$

Add the two ways

$$\frac{20}{72} + \frac{12}{72} = \frac{32}{72}$$

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$$\frac{32}{72} = \frac{4}{9}$$

c) Only look at the outcomes with the same year group

$$P(\text{both Year 11 given same}) = \frac{12}{72} \div \frac{32}{72}$$

$$P(\text{both Year 11 given same}) = \frac{12}{32}$$

$$P(\text{both Year 11 given same}) = \frac{3}{8}$$

a) $\frac{4}{8}, \frac{4}{8}, \frac{5}{8}, \frac{3}{8}$ b) $\frac{4}{9}$ c) $\frac{3}{8}$

4 A tin contains n biscuits. 3 of the biscuits are chocolate and the rest are plain. Zara takes two biscuits from the tin at random and eats them. The probability that both of her biscuits are chocolate is $\frac{1}{12}$

a. Show that $n(n - 1) = 72$ (2)

b. Find the value of n, and work out the probability that both of Zara's biscuits are plain. (3)

(5 marks)

a) After one chocolate biscuit is eaten

there are 2 chocolate biscuits left out of $n - 1$

$$P(\text{both chocolate}) = \frac{3}{n} \times \frac{2}{n-1}$$

$$\frac{6}{n(n-1)} = \frac{1}{12}$$

$$n(n-1) = 6 \times 12$$

$$n(n-1) = 72$$

b) Find two numbers next to each other that multiply to 72

$$9 \times 8 = 72$$

$$n = 9$$

Number of plain biscuits

$$9 - 3 = 6$$

$$P(\text{both plain}) = \frac{6}{9} \times \frac{5}{8}$$

$$P(\text{both plain}) = \frac{30}{72}$$

$$P(\text{both plain}) = \frac{5}{12}$$

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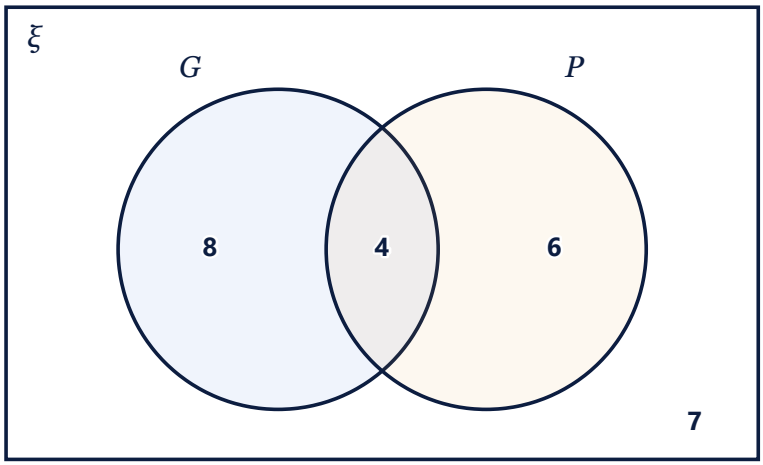
a) $n(n-1) = 6 \times 12 = 72$ b) $n = 9, \frac{5}{12}$

5 The Venn diagram shows how many of the 25 students in a music class play the guitar (G) and how many play the piano (P).

a. A student who plays the piano is picked at random. Write down the probability that this student also plays the guitar. (1)

b. Two of the 25 students are picked at random, without replacement. Work out the probability that both of them play the guitar. (2)

c. Given that both of the students picked play the guitar, work out the probability that both of them also play the piano. (2)



(5 marks)

a) Only look at the students who play the piano

$$4 + 6 = 10$$

$$P(\text{guitar given piano}) = \frac{4}{10}$$

$$P(\text{guitar given piano}) = \frac{2}{5}$$

b) Number who play the guitar

$$8 + 4 = 12$$

$$P(\text{both play guitar}) = \frac{12}{25} \times \frac{11}{24}$$

$$P(\text{both play guitar}) = \frac{132}{600}$$

$$P(\text{both play guitar}) = \frac{11}{50}$$

c) 4 students play both instruments

$$P(\text{both play both}) = \frac{4}{25} \times \frac{3}{24}$$

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$$P(\text{both play both}) = \frac{12}{600}$$

Divide by the probability that both play the guitar

$$\frac{12}{600} \div \frac{132}{600} = \frac{12}{132}$$

$$\frac{12}{132} = \frac{1}{11}$$

a) $\frac{2}{5}$ b) $\frac{11}{50}$ c) $\frac{1}{11}$

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