Normal distribution and control limits

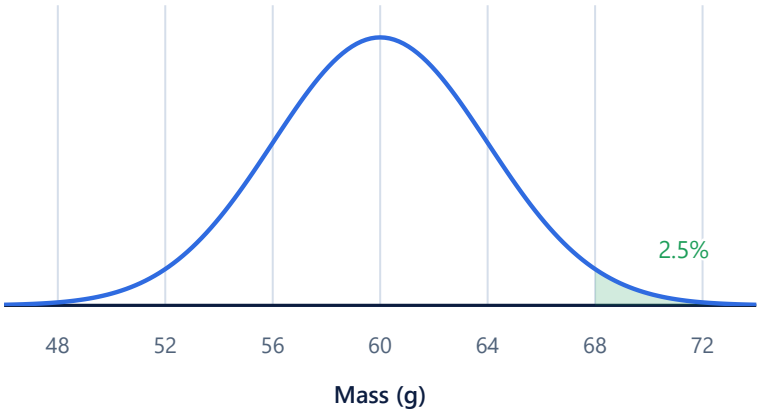
Grade 9 | Worked solutions

GCSE Statistics topic practice

1 The masses of the eggs from a farm are normally distributed with a mean of 60 g and a standard deviation of 4 g. The diagram shows this distribution.

a. Write down the percentage of the eggs that have a mass between 56 g and 64 g. (1)

b. A tray holds 400 of these eggs. Work out an estimate for the number of eggs in the tray that have a mass of more than 68 g. (2)



(3 marks)

a) 56 and 64 are 1 sd either side of the mean

$60 - 4 = 56$

$60 + 4 = 64$

68% of the eggs

b) 68 is 2 sd above the mean

$60 + 2 \times 4 = 68$

95% are within 2 sd of the mean

$100\% - 95\% = 5\%$

Half of these are above 68 g

$5\% \div 2 = 2.5\%$

$0.025 \times 400 = 10$

a) 68% b) 10 eggs

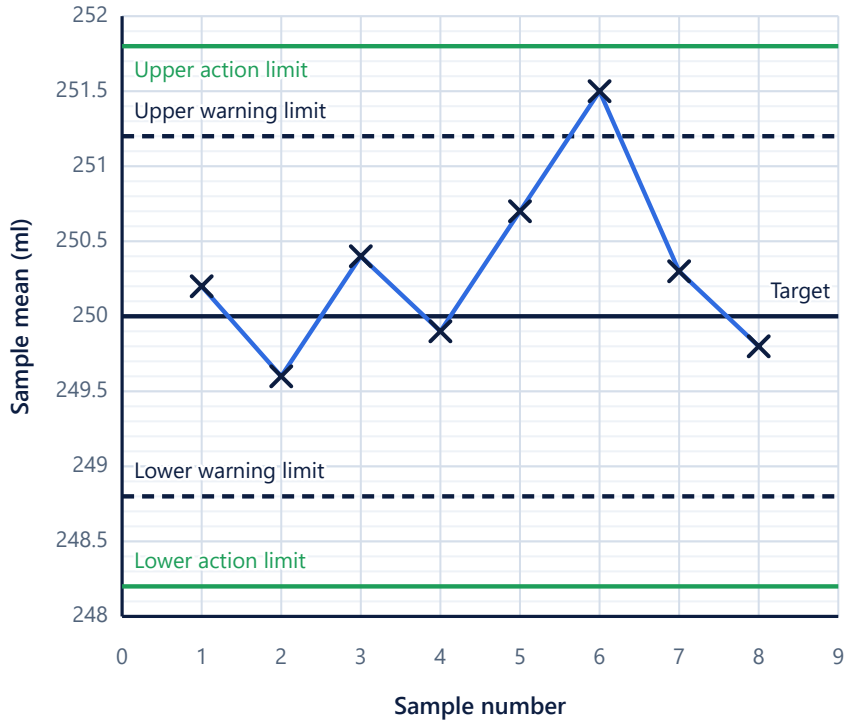
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2 A machine fills cartons with juice. The target for the mean volume of a sample of cartons is 250 ml. The control chart shows the target, the warning limits and the means of 8 samples. The warning limits are 2 standard deviations from the target.

a. Use the control chart to work out the standard deviation of the sample means. (1)

b. The action limits are 3 standard deviations from the target. Work out the action limits and draw them on the control chart. (2)

c. State what should have been done straight after sample 6 was taken. (1)



(4 marks)

a) The upper warning limit is at 251.2

251.2 - 250 = 1.2

This is 2 standard deviations

sd = 1.2 ÷ 2

sd = 0.6 ml

b) Action limits are 3 sd from the target

3 x 0.6 = 1.8

250 + 1.8 = 251.8

250 - 1.8 = 248.2

Draw horizontal lines at 248.2 and 251.8

c) Sample 6 is between the warning and action limits

Take another sample straight away

a) 0.6 ml b) 248.2 and 251.8 c) take another sample

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3 The lifetimes of a make of battery are normally distributed. 2.5% of the batteries last longer than 58 hours. 16% of the batteries last less than 43 hours.

a. Work out the mean and the standard deviation of the lifetimes. (3)

b. 2000 of these batteries are tested. Work out an estimate for the number that last less than 38 hours. (1)

(4 marks)

a) 95% are within 2 sd of the mean

so 2.5% are above mean + 2 sd

mean + 2 sd = 58

68% are within 1 sd of the mean

so 16% are below mean - 1 sd

mean - 1 sd = 43

Subtract the second equation from the first

3 sd = 15

sd = 5 hours

mean = 43 + 5

mean = 48 hours

b) 38 hours is 2 sd below the mean

48 - 2 x 5 = 38

2.5% of the batteries last less than 38 hours

0.025 x 2000 = 50

a) mean 48 hours, sd 5 hours b) 50 batteries

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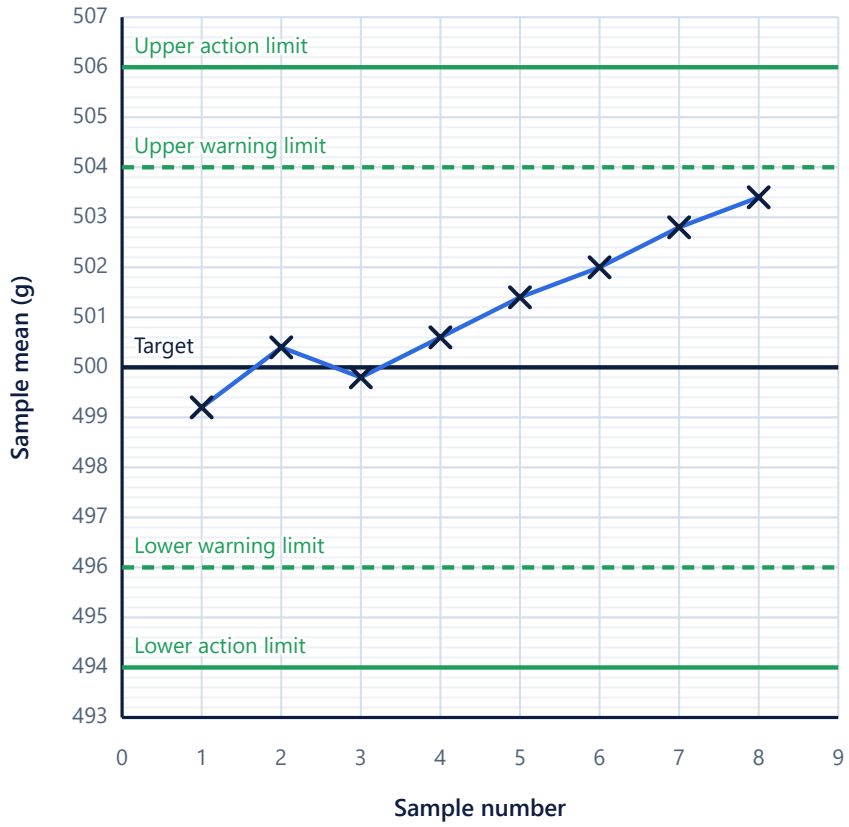
4 A machine fills bags with sugar. When the machine is working correctly, the mean masses of samples of bags are normally distributed with mean 500 g and standard deviation 2 g.

The warning limits on a control chart are set 2 standard deviations from the target, and the action limits are set 3 standard deviations from the target. The control chart shows the target line and the means of 8 samples.

a. Work out the warning limits and the action limits, and draw them on the control chart. (2)

b. When the machine is working correctly, what percentage of sample means would you expect to be outside the warning limits? (1)

c. Chidi says, "Every sample mean is inside the warning limits, so the machine does not need checking." Comment on Chidi's statement, using the control chart. (2)



(5 marks)

a) Warning limits are 2 sd from the target

$500 + 2 \times 2 = 504$

$500 - 2 \times 2 = 496$

Action limits are 3 sd from the target

$500 + 3 \times 2 = 506$

$500 - 3 \times 2 = 494$

Draw horizontal lines at 494, 496, 504 and 506

b) 95% of values are within 2 sd of the mean

$100\% - 95\% = 5\%$

c) Chidi is not right

The sample means have gone up five times in a row

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and the last five are all above the target

This trend shows the process is drifting

The machine should be checked now,

before a mean goes outside the limits

a) 496, 504 and 494, 506 b) 5% c) rising trend, check

5 A machine fills bags with crisps. The target for the mean mass of a sample of bags is 150 g. The warning limits are 2 standard deviations from the target and the action limits are 3 standard deviations from the target. The control chart shows the limits and the means of the first 5 samples. The table shows the means of the next 3 samples.

Sample number	6	7	8
Sample mean (g)	150.2	150.9	151.7

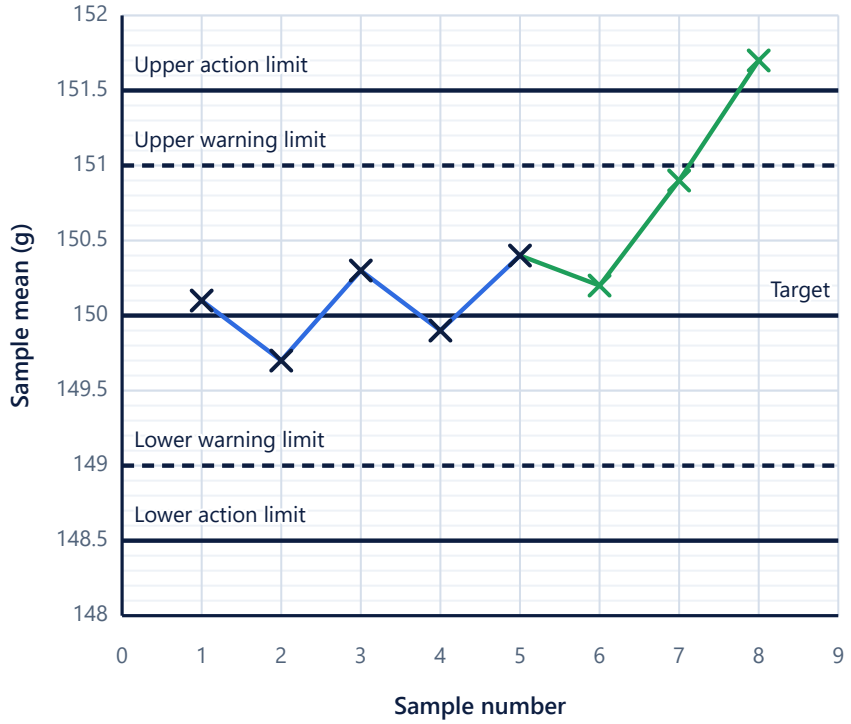
a. Plot the means of samples 6, 7 and 8 on the control chart. (1)

b. State what should be done after sample 8 is taken. (1)

c. When the machine is working correctly, the sample means are normally distributed. Out of 1000 samples, how many would you expect to have a mean outside the action limits? (1)

d. Give one reason why the company does not want the mean mass to be above the upper action limit. (1)

e. The machine is replaced by one that gives sample means with a smaller standard deviation. Describe how the limits on the control chart will change. (1)



(5 marks)

a) Plot (6, 150.2), (7, 150.9) and (8, 151.7)

Each small square on the mass axis is 0.1 g

b) 151.7 is above the upper action limit (151.5)

Stop the machine and reset it

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c) 99.7% of sample means are within 3 sd of the target

$100\% - 99.7\% = 0.3\%$

$0.003 \times 1000 = 3$

d)

The bags would hold more crisps than needed

This costs the company money

e) 2 sd and 3 sd will both be smaller

All four limits will be closer to the target

a) plotted b) stop, reset c) 3 d) cost e) closer

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