Sampling and capture-recapture assumptions

Grade 9 | Worked solutions

GCSE Statistics topic practice

1 A delivery company has 480 members of staff. The table shows the job each of them does.

Job	Office	Warehouse	Driver
Number of staff	240	150	90

The manager takes a sample stratified by job. There are 20 warehouse staff in her sample.

a. Work out the total number of staff in the sample. (2)

b. Work out the number of drivers in the sample. (1)

c. The manager wants to find out what the staff think about a new shift pattern. Give one reason why a stratified sample is better here than a simple random sample. (1)

(4 marks)

a) Find the fraction of the warehouse staff in the sample

$$\frac{20}{150} = \frac{2}{15}$$

The same fraction is taken from all the staff

$$\frac{2}{15} \times 480 = 64$$

b) Take the same fraction of the drivers

$$\frac{2}{15} \times 90 = 12$$

c)

Staff in different jobs may have different views

A stratified sample has each job in the right proportion

a) 64 b) 12 c) each job fairly represented

2 An ecologist catches 50 snails in a garden, marks them and releases them. The next day she catches 45 snails and some of them are marked. She uses her results to estimate that there are 750 snails in the garden.

a. Work out how many of the 45 snails in the second catch were marked. (2)

b. Work out what her estimate would have been if she had found one more marked snail in the second catch. Use your answer to comment on the reliability of her estimate of 750. (2)

(4 marks)

a) M = 50 marked, n = 45 caught later, m of these marked

$$N = \frac{M \times n}{m}$$

$$750 = \frac{50 \times 45}{m}$$

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$$750 = \frac{2250}{m}$$

$$m = \frac{2250}{750}$$

$$m = 3$$

b) One more marked snail gives m = 4

$$N = \frac{2250}{4}$$

$$N = 562.5$$

The estimate would be about 563

750 - 563 = 187

One snail changes the estimate by nearly 190

so the estimate of 750 is not reliable

a) 3 b) about 563, so not reliable

3 A ranger wants to estimate the number of fish in a lake. She catches 60 fish, tags them and puts them back. One week later she catches 80 fish and finds that 12 of them are tagged.

a. Work out an estimate for the number of fish in the lake. (2)

b. Write down one assumption the ranger has made. (1)

c. The ranger later finds out that 15 of the tagged fish died before the second catch, and that no other fish died, were born, or entered or left the lake. Work out a new estimate for the number of fish in the lake at the time of the second catch, and state whether the estimate in part (a) was an overestimate or an underestimate of this number. (2)

(5 marks)

a) M = 60 tagged, n = 80 caught later, m = 12 of these tagged

$$N = \frac{M \times n}{m}$$

$$N = \frac{60 \times 80}{12}$$

$$N = \frac{4800}{12}$$

$$N = 400$$

b) One of these

The tagged fish mixed fully with the other fish

No tags fell off

The population did not change between the catches

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c) Tagged fish alive at the second catch

$60 - 15 = 45$

$$N = \frac{45 \times 80}{12}$$

$$N = \frac{3600}{12}$$

$$N = 300$$

400 is greater than 300

so the estimate in part a) was an overestimate

a) 400 b) tagged fish mixed fully c) 300, overestimate

4 A researcher catches 40 voles in a field, marks each one with a dab of paint and releases them. A week later he catches 50 voles and 8 of them are marked. He estimates that there are 250 voles in the field.

For each of the following, state whether the true number of voles is likely to be greater than 250 or less than 250. Give a reason for each answer.

a. Heavy rain washed the paint off some of the marked voles before the second catch. (2)

b. The traps contained food, so voles that had been caught once were more likely to be caught again. (2)

(4 marks)

The estimate comes from $N = \frac{40 \times 50}{8}$

a) Less than 250

Some marked voles were counted as unmarked

so the 8 marked voles is too low

Dividing by a number that is too low

makes the estimate too high

b) Greater than 250

Too many marked voles were in the second catch

so the 8 marked voles is too high

Dividing by a number that is too high

makes the estimate too low

a) less than 250 b) greater than 250

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5 A council wants to know what the people living in its 12 000 households think about a change to bin collections. It has a list of all 12 000 households in order of address. An officer plans to telephone the first 200 households on the list on a Tuesday morning.

a. Give two reasons why the officer's sample is likely to be biased. (2)

b. Describe how the officer could use the list to take a systematic sample of 200 households. (2)

c. The households are in flats, terraced houses and detached houses. Give one advantage of taking a sample stratified by type of home instead of a systematic sample. (1)

(5 marks)

a) Reason 1

The first 200 addresses are from the same few streets so the rest of the area is not represented

Reason 2

People at work on a Tuesday morning will not answer so working people are under-represented

b) Find the gap between households

$12000 \div 200 = 60$

Number the households from 1 to 12000

Pick a random number from 1 to 60 for the first one

Then take every 60th household after it

c)

Each type of home is in the sample in the same proportion as in the whole council area

a) same streets, at work b) every 60th c) in proportion

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