

Highest common factor and lowest common multiple (listing)

Worked solutions

GCSE Maths topic practice



This topic online

- 1 Write down a factor of 20 that is greater than 5 and less than 20.

(1 mark)

Factors of 20: 1, 2, 4, 5, 10, 20

10

- 2 Write down all the factors of 18.

(2 marks)

$$1 \times 18 = 18$$

$$2 \times 9 = 18$$

$$3 \times 6 = 18$$

1, 2, 3, 6, 9, 18

- 3 Write down the lowest common multiple (LCM) of

3 6 9 18 54

(1 mark)

54 is a multiple of every number in the list

$$54 = 3 \times 18 = 6 \times 9 = 9 \times 6 = 18 \times 3 = 54 \times 1$$

Since 54 is itself in the list, no smaller number can work, so the LCM is 54

54

- 4 Write down all the factors of 30.

(2 marks)

$$1 \times 30$$

$$2 \times 15$$

$$3 \times 10$$

$$5 \times 6$$

1, 2, 3, 5, 6, 10, 15, 30

- 5 Find the highest common factor (HCF) of 54 and 90

(2 marks)

$$54 = 2 \times 3^3 \text{ and } 90 = 2 \times 3^2 \times 5$$

Take the lowest power of each common prime: 2 and 3^2

$$\text{HCF} = 2 \times 9 = 18$$

A common factor must divide both numbers, so it cannot use more copies of a prime than either number contains. That is why we choose the lower of the two powers

18

- 6 Find the lowest common multiple (LCM) of 6 and 8.

(2 marks)

Multiples of 6: 6, 12, 18, 24

Multiples of 8: 8, 16, 24

24

- 7 Find the highest common factor (HCF) of 48 and 84

(2 marks)

$$48 = 2^4 \times 3 \text{ and } 84 = 2^2 \times 3 \times 7$$

Take the lowest power of each common prime: 2^2 and 3

$$\text{HCF} = 4 \times 3 = 12$$

12

- 8 Write down all the numbers that are factors of both 24 and 40.

(2 marks)

24: 1, 2, 3, 4, 6, 8, 12, 24

40: 1, 2, 4, 5, 8, 10, 20, 40

1, 2, 4, 8

- 9 Find the lowest common multiple (LCM) of 9 and 12.

(2 marks)

9: 9, 18, 27, 36

12: 12, 24, 36

36

- 10 Write down two numbers, both greater than 20, whose highest common factor (HCF) is 7

(2 marks)

Both numbers must be multiples of 7 that are greater than 20
The multiples of 7 used must not share any other common factor
For example $21 = 3 \times 7$ and $28 = 4 \times 7$: since 3 and 4 have no common factor greater than 1, $HCF(21, 28) = 7$

- 11 Find the lowest common multiple (LCM) of 15 and 18

(3 marks)

$$15 = 3 \times 5 \text{ and } 18 = 2 \times 3^2$$

Take the highest power of every prime that appears: 2, 3^2 and 5

$$LCM = 2 \times 9 \times 5 = 90$$

A common multiple must contain enough prime factors to be divisible by both numbers.
Taking the larger power of each prime provides exactly enough, giving the lowest such multiple

90

- 12 Find the highest common factor (HCF) of 28 and 42.

(3 marks)

28: 1, 2, 4, 7, 14, 28

42: 1, 2, 3, 6, 7, 14, 21, 42

14

- 13 List all the factors of 28.

(2 marks)

Work in factor pairs: 1×28 , 2×14 , 4×7

The factors of 28 are 1, 2, 4, 7, 14 and 28

1, 2, 4, 7, 14, 28

- 14 A bus to the station leaves every 10 minutes. A bus to the market leaves every 25 minutes.

Both buses leave the bus stop together at 8:00 am.

At what time do they next leave together?

(3 marks)

10: 10, 20, 30, 40, 50

25: 25, 50

LCM = 50 minutes

8:50 am

15 Nadia is making table decorations.

Each decoration needs one candle and one holder.

Candles are sold in packs of 9

Holders are sold in packs of 12

Nadia buys exactly the same number of candles as holders, with none left over.

Work out the smallest number of packs of candles and the smallest number of packs of holders she can buy.

(3 marks)

The number of candles must equal the number of holders, so it must be a common multiple of 9 and 12

$$9 = 3^2 \text{ and } 12 = 2^2 \times 3, \text{ so LCM} = 2^2 \times 3^2 = 36$$

36 candles need $36 \div 9 = 4$ packs, and 36 holders need $36 \div 12 = 3$ packs

16 Ravi has 48 red counters and 36 blue counters.

He puts them into identical bags. Each bag has the same number of red counters and the same number of blue counters. No counters are left over.

What is the greatest number of bags Ravi can make?

(3 marks)

48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Highest common factor

12 bags

17 Two warning buoys are anchored in a harbour.

One buoy flashes every 24 seconds.

The other buoy flashes every 40 seconds.

At 8 pm the two buoys flash at the same time.

Work out how many seconds later the two buoys next flash at the same time.

(3 marks)

The buoys flash together at common multiples of 24 and 40

$$24 = 2^3 \times 3 \text{ and } 40 = 2^3 \times 5, \text{ so LCM} = 2^3 \times 3 \times 5 = 120$$

They next flash together 120 seconds (2 minutes) later

120 seconds

18 Work out how many positive whole numbers less than 100 are multiples of both 4 and 6.

(3 marks)

We count positive whole numbers only, excluding zero and negative multiples. The least common multiple of 4 and 6 is 12 because it is the first number in both positive multiplication lists

A number that is a multiple of both 4 and 6 must be a multiple of 12

Multiples of 12 less than 100: 12, 24, 36, 48, 60, 72, 84, 96

There are 8 such numbers

8

19 a. Find the highest common factor (HCF) of 72 and 120

b. Find the lowest common multiple (LCM) of 72 and 120

(4 marks)

$$72 = 2^3 \times 3^2 \text{ and } 120 = 2^3 \times 3 \times 5$$

a) HCF

Lowest powers of the common primes are 2^3 and 3, so $\text{HCF} = 8$

$\times$

3

=

24

b) LCM

highest powers of all primes are 2^3 , 3^2 and 5, so $\text{LCM} = 8 \times 9$

$\times 5 =$

360