



- 1 Write down the lowest common multiple (LCM) of

3 6 9 18 54

(1 mark)

54 is a multiple of every number in the list

$$54 = 3 \times 18 = 6 \times 9 = 9 \times 6 = 18 \times 3 = 54 \times 1$$

Since 54 is itself in the list, no smaller number can work, so the LCM is 54

54

- 2 Find the lowest common multiple (LCM) of 8 and 12.

(2 marks)

Multiples of 8: 8, 16, 24, 32

Multiples of 12: 12, 24, 36

$$\text{LCM} = 24$$

- 3 A bus to Hull leaves a station every 12 minutes. A bus to York leaves the same station every 18 minutes. Both buses leave at 08:00.

At what time do they next leave together?

(2 marks)

$$12 = 2^2 \times 3$$

$$18 = 2 \times 3^2$$

$$\text{LCM} = 2^2 \times 3^2 = 36$$

08:00 + 36 minutes

08:36

4

- Write 84 as a product of its prime factors.
- Write 126 as a product of its prime factors.
- Find the highest common factor (HCF) of 84 and 126.
- Find the lowest common multiple (LCM) of 84 and 126.

(5 marks)

a)

$$84 \rightarrow 2 \times 42 \rightarrow 2 \times 2 \times 21 \rightarrow 2 \times 2 \times 3 \times 7$$

$$84 = 2^2 \times 3 \times 7$$

b)

$$126 \rightarrow 2 \times 63 \rightarrow 2 \times 3 \times 21 \rightarrow 2 \times 3 \times 3 \times 7$$

$$126 = 2 \times 3^2 \times 7$$

c)

Common to both: 2, 3, 7 (lowest powers)

$$2 \times 3 \times 7 = 42$$

$$\text{HCF} = 42$$

d)

Highest powers: 2^2 , 3^2 , 7

$$4 \times 9 \times 7 = 252$$

$$\text{LCM} = 252$$

5

$$A = 2^3 \times 3^2 \times 5 \quad \text{and} \quad B = 2^2 \times 3 \times 7$$

- Find the highest common factor of A and B.
- Find the lowest common multiple of A and B.

Give both answers as ordinary numbers.

(3 marks)

a)

In both: 2 (lowest power 2^2) and 3 (lowest power 3)

$$2^2 \times 3 = 12$$

$$\text{HCF} = 12$$

b)

Highest powers: 2^3 , 3^2 , 5, 7

$$8 \times 9 \times 5 \times 7 = 2520$$

$$\text{LCM} = 2520$$

6 Find the highest common factor (HCF) of 54 and 90

(2 marks)

$$54 = 2 \times 3^3 \text{ and } 90 = 2 \times 3^2 \times 5$$

Take the lowest power of each common prime: 2 and 3^2

$$\text{HCF} = 2 \times 9 = 18$$

A common factor must divide both numbers, so it cannot use more copies of a prime than either number contains. That is why we choose the lower of the two powers

18

7 The prime factors of 60 and 90 have been sorted into a Venn diagram, shown here as a table.

Only in 60	In both	Only in 90
2	2 3 5	3

a. Use the diagram to find the HCF of 60 and 90.

b. Use the diagram to find the LCM of 60 and 90.

(3 marks)

a)

Overlap: 2, 3, 5

$$2 \times 3 \times 5 = 30$$

$$\text{HCF} = 30$$

b)

Multiply every number in the whole diagram

$$2 \times 2 \times 3 \times 5 \times 3 = 180$$

$$\text{LCM} = 180$$

8 Find the highest common factor (HCF) of 48 and 84

(2 marks)

$$48 = 2^4 \times 3 \text{ and } 84 = 2^2 \times 3 \times 7$$

Take the lowest power of each common prime: 2^2 and 3

$$\text{HCF} = 4 \times 3 = 12$$

12

- 9 The highest common factor of two numbers is 12 and their lowest common multiple is 72. One of the numbers is 36.

Work out the other number.

(3 marks)

$$36 = 2^2 \times 3^2$$

$$12 = 2^2 \times 3$$

$$72 = 2^3 \times 3^2$$

LCM has 2^3 , so the other number has 2^3

HCF has only one 3, so the other has one 3

$$2^3 \times 3 = 24$$

Check: $\text{HCF}(36, 24) = 12$ and $\text{LCM} = 72$ ✓

24

- 10 Write down two numbers, both greater than 20, whose highest common factor (HCF) is 7

(2 marks)

Both numbers must be multiples of 7 that are greater than 20

The multiples of 7 used must not share any other common factor

For example $21 = 3 \times 7$ and $28 = 4 \times 7$: since 3 and 4 have no common factor greater than 1, $\text{HCF}(21, 28) = 7$

- 11 Find the lowest common multiple (LCM) of 15 and 18

(3 marks)

$$15 = 3 \times 5 \text{ and } 18 = 2 \times 3^2$$

Take the highest power of every prime that appears: 2, 3^2 and 5

$$\text{LCM} = 2 \times 9 \times 5 = 90$$

A common multiple must contain enough prime factors to be divisible by both numbers. Taking the larger power of each prime provides exactly enough, giving the lowest such multiple

90

12 $A = 2^3 \times 3 \times 5^2$

$$B = 2^2 \times 3^2 \times 5$$

- Find the highest common factor (HCF) of A and B .
- Find the lowest common multiple (LCM) of A and B .
Give your answers as products of powers of primes.

(3 marks)

a) For the HCF take the lowest power of each common prime

$$2^2, 3 \text{ and } 5, \text{ so } \text{HCF} = 2^2 \times 3 \times 5 = 60$$

b) For the LCM take the highest power of each prime

$$2^3, 3^2 \text{ and } 5^2, \text{ so } \text{LCM} = 2^3 \times 3^2 \times 5^2 = 1800$$

13 Nadia is making table decorations.

Each decoration needs one candle and one holder.

Candles are sold in packs of 9

Holders are sold in packs of 12

Nadia buys exactly the same number of candles as holders, with none left over.

Work out the smallest number of packs of candles and the smallest number of packs of holders she can buy.

(3 marks)

The number of candles must equal the number of holders, so it must be a common multiple of 9 and 12

$$9 = 3^2 \text{ and } 12 = 2^2 \times 3, \text{ so } \text{LCM} = 2^2 \times 3^2 = 36$$

36 candles need $36 \div 9 = 4$ packs, and 36 holders need $36 \div 12 = 3$ packs

14 Two warning buoys are anchored in a harbour.

One buoy flashes every 24 seconds.

The other buoy flashes every 40 seconds.

At 8 pm the two buoys flash at the same time.

Work out how many seconds later the two buoys next flash at the same time.

(3 marks)

The buoys flash together at common multiples of 24 and 40

$$24 = 2^3 \times 3 \text{ and } 40 = 2^3 \times 5, \text{ so } \text{LCM} = 2^3 \times 3 \times 5 = 120$$

They next flash together 120 seconds (2 minutes) later

120 seconds

15

- a. Write 126 as a product of its prime factors.
b. Find the lowest common multiple (LCM) of 126 and 60

(4 marks)

$$a) 126 = 2 \times 63, 63 = 9 \times 7 = 3^2 \times 7, \text{ so } 126 = 2 \times 3^2 \times 7$$

$$b) 60 = 2^2 \times 3 \times 5$$

b) Take the highest power of each prime

$$2^2, 3^2, 5 \text{ and } 7, \text{ so } LCM = 4 \times 9 \times 5 \times 7 = 1260$$

16

- a. Write 448 as a product of powers of its prime factors.
b. Find the highest common factor (HCF) of 448 and 168

(4 marks)

$$a) 448 = 2 \times 224 = 2 \times 2 \times 112 = 2 \times 2 \times 2 \times 56 = 2 \times 2 \times 2 \times 2 \times 28 = 2 \times 2 \times 2 \times 2 \times 2 \times 14 = 2^6 \times 7$$

$$b) 168 = 2^3 \times 3 \times 7$$

b) Take the lowest power of each common prime

$$2^3 \text{ and } 7, \text{ so } HCF = 8 \times 7 = 56$$

17

- a. Find the highest common factor (HCF) of 72 and 120
b. Find the lowest common multiple (LCM) of 72 and 120

(4 marks)

$$72 = 2^3 \times 3^2 \text{ and } 120 = 2^3 \times 3 \times 5$$

a) HCF

$$\text{lowest powers of the common primes are } 2^3 \text{ and } 3, \text{ so } HCF = 8$$

$$\times$$

$$3 =$$

$$24$$

b) LCM

$$\text{highest powers of all primes are } 2^3, 3^2 \text{ and } 5, \text{ so } LCM = 8 \times 9$$

$$\times 5 =$$

$$360$$