

Comparing data using averages and spread

Worked solutions

GCSE Maths topic practice



This topic online

- 1 A shoe shop records the sizes of the last eight pairs of trainers it sold.

5 7 6 5 8 5 9 6

- a. Write down the modal shoe size.
b. Work out the range of the shoe sizes.

(3 marks)

a) Size 5 appears three times, more than any other size, so the mode is 5

b) Range

$$9 - 5 = 4$$

2

A hockey team played 25 matches. The table shows the number of goals the team scored in each match.

Goals scored	Frequency
0	4
1	7
2	9
3	3
4	2

- Write down the modal number of goals scored.
- Work out the total number of goals the team scored in the 25 matches.

(3 marks)

a)

Highest frequency is 9

mode = 2 goals

b)

goals $\times$ frequency

$$0 \times 4 = 0$$

$$1 \times 7 = 7$$

$$2 \times 9 = 18$$

$$3 \times 3 = 9$$

$$4 \times 2 = 8$$

$$0 + 7 + 18 + 9 + 8 = 42$$

42 goals

3

The table shows the number of children in each of 20 families.

Number of children	Frequency
0	3
1	8
2	6
3	3

Work out the mean number of children per family.

(3 marks)

$$\text{Total children} = (0 \times 3) + (1 \times 8) + (2 \times 6) + (3 \times 3) = 0 + 8 + 12 + 9 = 29$$

$$\text{Total families} = 20$$

$$\text{Mean} = 29 \div 20 = 1.45$$

1.45

4

24 pupils were asked how many books they read in the summer holidays.

Number of books	Frequency
0	5
1	6
2	7
3	4
4	2

Dev says, "The median is 7 because 7 is the middle frequency."

- Explain why Dev is wrong and work out the median number of books.
- Work out the total number of books the 24 pupils read.

(4 marks)

a)

7 is a frequency (pupils), not a number of books

24 pupils: median is between the 12th and 13th

5 read 0 books (up to 5th)

+ 6 read 1 book (up to 11th)

+ 7 read 2 books (up to 18th)

12th and 13th are both 2

median = 2 books

b)

$$0 \times 5 + 1 \times 6 + 2 \times 7 + 3 \times 4 + 4 \times 2$$

$$0 + 6 + 14 + 12 + 8 = 40$$

40 books

5

A small firm has six employees. Their yearly salaries are
£22 000 £24 000 £25 000 £25 000 £27 000 £97 000

The firm wants to advertise a job and tell applicants the "typical" salary.

- Work out the mean salary and the median salary.
- Which average should the firm use to describe the typical salary? Give a reason for your answer. (4 marks)

a)

$$\text{Total} = 22 + 24 + 25 + 25 + 27 + 97 = 220 \text{ (thousand)}$$

$$\text{Mean} = 220\,000 \div 6 = \text{£}36\,666.67$$

$$\text{mean} \approx \text{£}36\,667$$

Middle two values are 25 000 and 25 000

$$\text{median} = \text{£}25\,000$$

b)

£97 000 is an outlier, it pulls the mean up

5 out of 6 people earn less than the mean

$$\text{Use the median (£25 000)}$$

6

There are 10 boys and 15 girls in a class.

The mean score of the 10 boys in a test was 6.2.

The mean score of the 15 girls in the same test was 5.8.

Mr Field says, "The mean score for the whole class was 6."

Show that Mr Field is wrong. (3 marks)

The groups have different sizes, so their means cannot be given equal weight.

Recover each group's total first, then divide the combined total by the combined number of people

Total for boys

$$10 \times 6.2 = 62$$

Total for girls

$$15 \times 5.8 = 87$$

Whole-class total

$$62 + 87 = 149, \text{ and } 10 + 15 = 25 \text{ students}$$

Mean

$$149 \text{ divided by } 25 = 5.96, \text{ which is not } 6, \text{ so Mr Field is wrong}$$

$$\text{Whole-class mean} = 5.96, \text{ not } 6$$

7

The table shows the number of items bought by each of 40 customers at a corner shop.

Number of items	Frequency
1	12
2	15
3	9
4	4

- a. Work out the mean number of items bought per customer.
 b. Kai says, "The mode is 15." Explain the mistake Kai has made.

(4 marks)

$$\text{Total items} = (1 \times 12) + (2 \times 15) + (3 \times 9) + (4 \times 4) = 12 + 30 + 27 + 16 = 85$$

$$\text{Mean} = 85 \div 40 = 2.125$$

The mode is the number of items with the highest frequency, so the mode is 2, not 15

8

Two classes, A and B, sat the same maths test. The five-figure summaries of their marks are shown in the table.

	Lowest	Lower quartile	Median	Upper quartile	Highest
Class A	12	25	38	46	60
Class B	20	30	38	40	52

- a. Work out the interquartile range for each class.
 b. Compare the marks of the two classes. Refer to the median and the interquartile range in your answer.

(4 marks)

a)

$$\text{IQR} = \text{UQ} - \text{LQ}$$

$$\text{Class A: } 46 - 25 = 21$$

$$\text{Class B: } 40 - 30 = 10$$

b)

Medians are both 38

So the typical mark is the same

B has the smaller IQR ($10 < 21$)

Same average, but Class B is more consistent

9

Nine students in Year 7 and nine students in Year 11 ran a race. Here are their times, in seconds.

Year 7: 12 15 15 18 20 21 24 26 28

Year 11: 8 10 13 14 16 19 22 25 27

Compare the times of the Year 7 students with the times of the Year 11 students.

(4 marks)

Year 7 median

the 5th value = 20 seconds. Year 11 median: the 5th value = 16 seconds

Year 7 range

28 - 12 = 16 seconds. Year 11 range: 27 - 8 = 19 seconds

The Year 11 median is lower, so on average the Year 11 students ran faster

The Year 7 range is smaller, so the Year 7 times were more consistent

10

Two cafés each recorded the waiting times, in minutes, of seven customers.

Café X: 4 5 5 6 7 8 9

Café Y: 3 4 4 5 5 6 25

- Work out the mean and the range of the waiting times for each café.
- Zara says, "Café X is better because its mean waiting time is lower." Work out the median for each café and explain why the median gives a fairer comparison here.

(5 marks)

a)

$$X: 4+5+5+6+7+8+9 = 44$$

$$\text{Mean} = 44 \div 7 = 6.3 \text{ (1 dp)}$$

$$\text{Range} = 9 - 4 = 5$$

$$Y: 3+4+4+5+5+6+25 = 52$$

$$\text{Mean} = 52 \div 7 = 7.4 \text{ (1 dp)}$$

$$\text{Range} = 25 - 3 = 22$$

b)

Median = 4th value of 7

X median = 6

Y median = 5

25 is an outlier, it pulls the mean of Y up

Median: a typical customer waits less at Y

11

The table shows information about the times, t seconds, taken by 40 swimmers to complete one length of a pool.

Time (t seconds)	Frequency
$20 < t \leq 30$	8
$30 < t \leq 40$	15
$40 < t \leq 50$	12
$50 < t \leq 60$	5

- a. Work out an estimate for the mean time.
 b. Explain why your answer to part (a) is an estimate.

(4 marks)

The individual measurements are unknown. Represent each class by its midpoint, halfway between its boundaries, then multiply by its frequency to estimate that class's total. Divide the sum of these totals by the total frequency

Use midpoints 25, 35, 45, 55

$$(25 \times 8) + (35 \times 15) + (45 \times 12) + (55 \times 5) = 200 + 525 + 540 + 275 = 1540$$

$$\text{Estimated mean} = 1540 \div 40 = 38.5 \text{ seconds}$$

The mean is an estimate because the individual times within each class are unknown, so midpoints are used to represent them

12

The table shows information about the distances, d km, that 40 employees travel to work.

Distance (d km)	Frequency
$0 < d \leq 5$	9
$5 < d \leq 10$	16
$10 < d \leq 15$	11
$15 < d \leq 20$	4

- Work out an estimate for the mean distance travelled.
- Write down the modal class.
- Work out the fraction of the employees who travel more than 10 km. Give your fraction in its simplest form.

(5 marks)

The individual measurements are unknown. Represent each class by its midpoint, halfway between its boundaries, then multiply by its frequency to estimate that class's total. Divide the sum of these totals by the total frequency

Use midpoints 2.5, 7.5, 12.5, 17.5

$$(2.5 \times 9) + (7.5 \times 16) + (12.5 \times 11) + (17.5 \times 4) = 22.5 + 120 + 137.5 + 70 = 350$$

$$\text{Estimated mean} = 350 \div 40 = 8.75 \text{ km}$$

The modal class is the class with the highest frequency: $5 < d \leq 10$

More than 10 km

$$11 + 4 = 15 \text{ employees, so the fraction is } \frac{15}{40} = \frac{3}{8}$$