



- 1  $a$  and  $b$  are two numbers in the ratio  $a : b = 3 : 5$

$$b - a = 12$$

Work out the value of  $a$  and the value of  $b$ .

(3 marks)

$$a = 3n, b = 5n$$

$$5n - 3n = 12$$

$$2n = 12$$

$$\div 2 \quad n = 6 \quad \div 2$$

$$a = 3 \times 6 = 18$$

$$b = 5 \times 6 = 30$$

$$a = 18, b = 30$$

- 2 The ratio  $x : (x + 4)$  is equal to  $3 : 5$

Work out the value of  $x$ .

(3 marks)

$$\frac{x}{(x+4)} = \frac{3}{5}$$

$$5x = 3(x + 4)$$

$$5x = 3x + 12$$

$$-3x \quad 2x = 12 \quad -3x$$

$$\div 2 \quad x = 6 \quad \div 2$$

$$\text{Check: } 6 : 10 = 3 : 5$$

$$x = 6$$

3

A bag contains only red, blue and yellow counters.

The numbers of red, blue and yellow counters are in the ratio  $2 : 3 : x$ .

A counter is taken at random from the bag.

The probability that it is yellow is  $\frac{2}{7}$ .

Work out the value of  $x$ .

(3 marks)

$$\text{Fraction of counters that are yellow} = \frac{x}{2 + 3 + x} = \frac{x}{5 + x}$$

$$\text{So } \frac{x}{5 + x} = \frac{2}{7}$$

$$7x = 2(5 + x), \text{ so } 7x = 10 + 2x, \text{ giving } 5x = 10 \text{ and } x = 2$$

Check: ratio  $2 : 3 : 2$  has total 7 parts and yellow is  $\frac{2}{7}$

2

4

A jar contains only red sweets and green sweets.

The ratio of the number of red sweets to the number of green sweets is  $4 : 3$ .

Show that the total number of sweets in the jar must be a multiple of 7.

(3 marks)

Let the number of red sweets be  $4n$  and the number of green sweets be  $3n$ , where  $n$  is a whole number

$$\text{Total} = 4n + 3n = 7n$$

$7n$  is always a multiple of 7, so the total must be a multiple of 7

5

Zainab, Yusuf and Ola share £720 in the ratio 5 : 3 : 4

- Work out how much Zainab gets.
- Yusuf gives one third of his share to Ola. Write down the new ratio of the amounts that Zainab, Yusuf and Ola have, in its simplest form.
- Work out the new amount that Ola has as a percentage of the £720. Give your answer correct to 1 decimal place.

(5 marks)

a)

$$5 + 3 + 4 = 12 \text{ parts}$$

$$720 \div 12 = 60$$

$$\text{Zainab: } 5 \times 60 = £300$$

$$\boxed{£300}$$

b)

$$\text{Yusuf: } 3 \times 60 = £180, \text{ Ola: } 4 \times 60 = £240$$

$$\frac{1}{3} \text{ of } 180 = 60$$

$$\text{Yusuf: } 180 - 60 = £120$$

$$\text{Ola: } 240 + 60 = £300$$

$$300 : 120 : 300 (\div 60)$$

$$\boxed{5 : 2 : 5}$$

c)

$$\frac{300}{720} \times 100$$

$$\boxed{41.7\%}$$

- 6 The ratio  $(2x + 1) : (x + 5)$  is equal to  $3 : 2$

Work out the value of  $x$ .

(3 marks)

$$\frac{(2x+1)}{(x+5)} = \frac{3}{2}$$

$$\begin{array}{rcl} 2(2x+1) & = & 3(x+5) \\ 4x+2 & = & 3x+15 \\ -3x & & -3x \\ x+2 & = & 15 \\ -2 & & -2 \\ x & = & 13 \end{array}$$

Check:  $27 : 18 = 3 : 2$

$$x = 13$$

- 7 The ratio of the amount of money Amy has to the amount Ben has is  $5 : 3$ .

Amy gives Ben £14.

The ratio of the amount Amy has to the amount Ben has is now  $6 : 5$ .

Work out how much money Amy had at first.

(4 marks)

Let Amy have  $5x$  pounds and Ben have  $3x$  pounds at first

After the gift

$$(5x - 14) : (3x + 14) = 6 : 5$$

$$\text{So } 5(5x - 14) = 6(3x + 14), \text{ giving } 25x - 70 = 18x + 84$$

$$7x = 154, \text{ so } x = 22 \text{ and Amy had } 5 \times 22 = \text{£}110$$

Check

$$110 - 14 = 96 \text{ and } 66 + 14 = 80, \text{ and } 96 : 80 = 6 : 5$$

$$\text{£}110$$

8

The ages of a mother and her daughter are in the ratio 7 : 2

In 5 years' time, the ratio of their ages will be 8 : 3

Work out how old the mother and daughter are now.

(4 marks)

Now:  $7k$  and  $2k$

$$\frac{(7k+5)}{(2k+5)} = \frac{8}{3}$$

$$3(7k + 5) = 8(2k + 5)$$

$$21k + 15 = 16k + 40$$

$$\begin{array}{rcl} -16k & 5k + 15 & = 40 \end{array} \quad \begin{array}{rcl} -16k & & \end{array}$$

$$\begin{array}{rcl} -15 & 5k & = 25 \end{array} \quad \begin{array}{rcl} -15 & & \end{array}$$

$$\begin{array}{rcl} \div 5 & k & = 5 \end{array} \quad \begin{array}{rcl} \div 5 & & \end{array}$$

Mother  $7 \times 5 = 35$ , daughter  $2 \times 5 = 10$

Check:  $40 : 15 = 8 : 3$

35 and 10

9

The cost of a pencil and the cost of a ruler are in the ratio 2 : 5.

Priti buys 3 pencils and 2 rulers for a total of £3.20.

Work out the cost of a ruler.

(4 marks)

Let a pencil cost  $2k$  pence and a ruler cost  $5k$  pence

$$3 \times 2k + 2 \times 5k = 16k = 320 \text{ pence}$$

$$\text{So } k = 20, \text{ and a ruler costs } 5 \times 20 = 100p = \text{£}1$$

Check

$$3 \times 40p + 2 \times \text{£}1 = \text{£}1.20 + \text{£}2 = \text{£}3.20$$

£1

10

The ratio of the amount of money Priya has to the amount of money Quinn has is 7 : 5.

Priya gives Quinn £18.

Now they each have the same amount of money.

Work out how much money Priya had originally.

(4 marks)

Let Priya have  $7x$  and Quinn have  $5x$

After the gift

$$7x - 18 = 5x + 18$$

$$2x = 36, \text{ so } x = 18$$

Priya originally had  $7 \times 18 = \text{£}126$

Check

$126 - 18 = 108$  and  $90 + 18 = 108$ , which are equal

£126

11

A box contains red counters and blue counters.

If 5 more red counters are put in the box, the ratio of red to blue will be 2 : 3.

If instead 5 more blue counters are put in the box, the ratio of red to blue will be 1 : 2.

Work out how many counters of each colour are in the box.

(5 marks)

Let there be  $r$  red and  $b$  blue counters

First condition

$$\frac{(r + 5)}{b} = \frac{2}{3}, \text{ so } 3r + 15 = 2b$$

Second condition

$$\left(b + 5\right) = \frac{r}{2}, \text{ so } 2r = b + 5, \text{ giving } b = 2r - 5$$

Substitute

$$3r + 15 = 2(2r - 5) = 4r - 10, \text{ so } r = 25 \text{ and } b = 45$$

Check

$$(25 + 5) : 45 = 30 : 45 = 2 : 3, \text{ and } 25 : (45 + 5) = 25 : 50 = 1 : 2$$

12

 $a$  and  $b$  are positive numbers such that

$$a^2 + 6b^2 = 5ab$$

Find the possible values of the ratio  $a : b$ .

(5 marks)

Rearrange

$$a^2 - 5ab + 6b^2 = 0$$

Factorise

$$(a - 2b)(a - 3b) = 0$$

So  $a = 2b$  or  $a = 3b$ Therefore  $a$ 

$$b = 2 : 1 \text{ or } a : b = 3 : 1$$

Check

$$\text{if } a = 2b \text{ then } a^2 + 6b^2 = 4b^2 + 6b^2 = 10b^2 \text{ and } 5ab = 10b^2; \text{ if } a = 3b \text{ then } 9b^2 + 6b^2 = 15b^2 \text{ and } 5ab = 15b^2$$

$$a : b = 2 : 1 \text{ or } a : b = 3 : 1$$

13

A box contains only red counters, blue counters and green counters.

The number of blue counters is twice the number of red counters.

The number of green counters is 9 more than the number of red counters.

There are 49 counters in the box altogether.

Work out the ratio of red counters to green counters. Give your answer in its simplest form.

(5 marks)

Let  $r$  be the number of red counters

$$\text{Blue} = 2r \text{ and green} = r + 9$$

$$r + 2r + r + 9 = 49$$

$$4r = 40, \text{ so } r = 10$$

There are 10 red, 20 blue and 19 green counters

Check

$$10 + 20 + 19 = 49$$

Ratio of red to green = 10 : 19, which is already in simplest form

$$10 : 19$$

14

In a box, the ratio of the number of red counters to the number of blue counters is 4 : 7.

12 more red counters are put into the box.

The ratio of the number of red counters to the number of blue counters is now 2 : 3.

Work out the number of blue counters in the box.

(4 marks)

Let there be  $4x$  red and  $7x$  blue counters at the start

After adding 12 red

$$(4x + 12) : 7x = 2 : 3$$

Cross multiply

$$3(4x + 12) = 2 \times 7x, \text{ so } 12x + 36 = 14x$$

$$2x = 36, \text{ so } x = 18$$

$$\text{Blue counters} = 7 \times 18 = 126$$

Check

$$\text{red} = 72 + 12 = 84, \text{ and } 84 : 126 = 2 : 3$$

|     |
|-----|
| 126 |
|-----|