



1 $f(x) = x^2 - 6x + 2$

a. Show that the equation $f(x) = 0$ has a root between $x = 0$ and $x = 1$

b. Show that the equation $x^2 - 6x + 2 = 0$ can be rearranged to give

$$x = \frac{x^2 + 2}{6}$$

(4 marks)

This polynomial has a continuous graph: it has no jumps or breaks. If its value changes from positive to negative, the graph must pass through zero somewhere in between

a) $f(0) = 0 - 0 + 2 = 2$, which is positive

a) $f(1) = 1 - 6 + 2 = -3$, which is negative

a) f is continuous and changes sign between 0 and 1, so there is a root between $x = 0$ and $x = 1$

b) Rearrange $x^2 - 6x + 2 = 0$ to $6x = x^2 + 2$

b) Divide both sides by 6

$$x = \frac{x^2 + 2}{6}, \text{ as required}$$

2 A square has an area of 70 cm^2 .

Use trial and improvement to find the length of a side, correct to 1 decimal place. You must show all your trials.

(3 marks)

$8.3^2 = 68.89$ too small

$8.4^2 = 70.56$ too big

between 8.3 and 8.4 - try the middle

$8.35^2 = 69.7225$ too small

so the side is between 8.35 and 8.4

8.4 cm

3

$$f(x) = x^3 + 4x - 9$$

- Show that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$
- Show that the equation $x^3 + 4x - 9 = 0$ can be rearranged to give

$$x = \sqrt[3]{9 - 4x}$$
- Starting with $x_0 = 1.5$, use the iteration formula $x_{n+1} = \sqrt[3]{9 - 4x_n}$ to work out the values of x_1 , x_2 and x_3 . Give each value correct to 4 decimal places.

(6 marks)

This polynomial has a continuous graph: it has no jumps or breaks. If its value changes from positive to negative, the graph must pass through zero somewhere in between

Keep the full calculator value for each next iteration (use the Ans key). Round only the values you write down, otherwise rounding errors can build up

$$a) f(1) = 1 + 4 - 9 = -4, \text{ which is negative}$$

$$a) f(2) = 8 + 8 - 9 = 7, \text{ which is positive}$$

a) f is continuous and changes sign, so there is a root between $x = 1$ and $x = 2$

b) Rearrange to $x^3 = 9 - 4x$, then take the cube root: $x = \text{cube root of } (9 - 4x)$, as required

$$c) x_1 = \text{cube root of } (9 - 4 \times 1.5) = \text{cube root of } 3 = 1.442250... = 1.4423 \text{ (4 dp)}$$

$$c) x_2 = \text{cube root of } (9 - 4 \times 1.442250) = 1.478356... = 1.4784 \text{ (4 dp)}$$

$$c) x_3 = \text{cube root of } (9 - 4 \times 1.478356) = 1.455992... = 1.4560 \text{ (4 dp)}$$

4

Use trial and improvement to find the cube root of 300, correct to 1 decimal place. Do not use the cube root key on your calculator. You must show all your trials.

(3 marks)

$$6.6^3 = 287.496 \text{ too small}$$

$$6.7^3 = 300.763 \text{ too big}$$

try the middle, 6.65

$$6.65^3 = 294.08 \text{ too small}$$

between 6.65 and 6.7

6.7

- 5 The equation $x^3 + 2x = 50$ has a solution between 3 and 4.

Use trial and improvement to find this solution correct to 1 decimal place. You must show all your trials.

(4 marks)

$x = 3.5$: 49.875 too small

$x = 3.6$: 53.856 too big

between 3.5 and 3.6 - try 3.55

$x = 3.55$: 51.839 too big

the solution is below 3.55

$x = 3.5$

- 6 The equation $x^3 - 3x = 10$ has a solution between 2 and 3.

Use trial and improvement to find this solution correct to 1 decimal place. You must show all your trials.

(4 marks)

$x = 2.6$: 9.776 too small

$x = 2.7$: 11.583 too big

try the middle, 2.65

$x = 2.65$: 10.660 too big

the solution is between 2.6 and 2.65

$x = 2.6$

- 7 A cuboid has a width of x cm, a length of $(x + 2)$ cm and a height of $(x + 5)$ cm. The volume of the cuboid is 200 cm^3 .

Use trial and improvement to find the value of x , correct to 1 decimal place. You must show all your trials.

(4 marks)

$x = 3.8$: $3.8 \times 5.8 \times 8.8 = 193.952$ too small

$x = 3.9$: $3.9 \times 5.9 \times 8.9 = 204.789$ too big

try the middle, 3.85

$x = 3.85$: 199.32 too small

between 3.85 and 3.9

$x = 3.9$

8

$$f(x) = x^2 + 2x - 5$$

a. Show that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$

b. Show that the equation $x^2 + 2x - 5 = 0$ can be rearranged to give

$$x = \frac{5}{x+2}$$

c. Starting with $x_0 = 1.5$, use the iteration formula $x_{n+1} = \frac{5}{x_n + 2}$ to work out the values of x_1, x_2 and x_3 . Give each value correct to 4 decimal places.

d. Write down an estimate of this root of $f(x) = 0$ correct to 2 decimal places.

(6 marks)

This polynomial has a continuous graph: it has no jumps or breaks. If its value changes from positive to negative, the graph must pass through zero somewhere in between

Keep the full calculator value for each next iteration (use the Ans key). Round only the values you write down, otherwise rounding errors can build up

a) $f(1) = 1 + 2 - 5 = -2$, which is negative, and $f(2) = 4 + 4 - 5 = 3$, which is positive

a) The sign change means there is a root between $x = 1$ and $x = 2$

b) Rearrange to $x^2 + 2x = 5$, factorise the left side as $x(x+2) = 5$, then divide by $(x+2)$: $x = \frac{5}{x+2}$, as required

$$c) x_1 = \frac{5}{1.5 + 2} = 1.428571... = 1.4286 \text{ (4 dp)}$$

$$c) x_2 = \frac{5}{1.428571 + 2} = 1.458333... = 1.4583 \text{ (4 dp)}$$

$$c) x_3 = \frac{5}{1.458333 + 2} = 1.445783... = 1.4458 \text{ (4 dp)}$$

d) The iterates settle around 1.45; the exact root is $-1 + \sqrt{6} = 1.449489...$, so 1.45 to 2 dp

To check the 2-decimal-place rounding, $f(1.445) = -0.021975$ and $f(1.455) = 0.027025$. The positive root lies between these rounding boundaries, so it rounds to 1.45

9

The equation $x^3 - x = 20$ has a solution between 2 and 3.

Use trial and improvement to find this solution correct to 2 decimal places. You must show all your trials.

(5 marks)

$x = 2.8$: 19.152 too small

$x = 2.9$: 21.489 too big

$x = 2.83$: 19.835 too small

$x = 2.84$: 20.066 too big

try the middle, 2.835

$x = 2.835$: 19.951 too small

the solution is between 2.835 and 2.84

$x = 2.84$