



- 1 Shape S is rotated 90° clockwise about $(0, 0)$ and the image is then rotated 180° about $(0, 0)$.

Describe fully the single transformation that maps S onto its final image.

(2 marks)

Same centre, so add the turns

$$90^\circ + 180^\circ = 270^\circ$$

270° clockwise = 90° anticlockwise

Rotation 90° anticlockwise, centre $(0, 0)$

- 2 Triangle A has vertices $(1, 1)$, $(3, 1)$ and $(1, 2)$.

Triangle A is rotated 90° clockwise about $(0, 0)$ to give triangle B .

Triangle B is then translated by the vector $(2, 3)$ to give triangle C .

Write down the coordinates of the vertices of triangle C .

(3 marks)

Rotate 90° clockwise: $(x, y) \rightarrow (y, -x)$

$$(1, 1) \rightarrow (1, -1)$$

$$(3, 1) \rightarrow (1, -3)$$

$$(1, 2) \rightarrow (2, -1)$$

Translate by $(2, 3)$: add 2 to x , add 3 to y

$$(1, -1) \rightarrow (3, 2)$$

$$(1, -3) \rightarrow (3, 0)$$

$$(2, -1) \rightarrow (4, 2)$$

$C: (3, 2), (3, 0), (4, 2)$

3

Triangle T has vertices $(1, 2)$, $(4, 2)$ and $(1, 4)$.

T is translated by the vector $(-3, -1)$ and the image is then reflected in the y -axis.

Write down the coordinates of the vertices of the final image.

(3 marks)

Translate by $(-3, -1)$

$$(1, 2) \rightarrow (-2, 1)$$

$$(4, 2) \rightarrow (1, 1)$$

$$(1, 4) \rightarrow (-2, 3)$$

Reflect in the y -axis: x changes sign

$$(-2, 1) \rightarrow (2, 1)$$

$$(1, 1) \rightarrow (-1, 1)$$

$$(-2, 3) \rightarrow (2, 3)$$

$$(2, 1), (-1, 1), (2, 3)$$

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Triangle T has vertices at $(1, 1)$, $(3, 1)$ and $(1, 2)$.

Triangle T is reflected in the y -axis to give triangle T_1 .

Triangle T_1 is then reflected in the x -axis to give triangle T_2 .

Describe fully the single transformation that maps triangle T onto triangle T_2 .

(3 marks)

Reflecting in the y -axis maps (x, y) to $(-x, y)$, so T_1 has vertices $(-1, 1)$, $(-3, 1)$, $(-1, 2)$

Reflecting T_1 in the x -axis maps (x, y) to $(x, -y)$, so T_2 has vertices $(-1, -1)$, $(-3, -1)$, $(-1, -2)$

Overall (x, y) has mapped to $(-x, -y)$, which is a rotation of 180° about the origin

So the single transformation is a rotation of 180° , centre $(0, 0)$

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Shape P is reflected in the line $x = 3$. The image is then reflected in the line $y = 2$.

- Write down the coordinates of the one point that is not moved by either reflection.
- Describe fully the single transformation that maps P onto its final image.

(3 marks)

a)

The point must lie on both lines

$$x = 3 \text{ and } y = 2$$

$$(3, 2)$$

b)

Test the point $(1, 1)$

$$\text{Reflect in } x = 3: (1, 1) \rightarrow (5, 1)$$

$$\text{Reflect in } y = 2: (5, 1) \rightarrow (5, 3)$$

$(3, 2)$ is the midpoint of $(1, 1)$ and $(5, 3)$

$$\text{Rotation } 180^\circ \text{ about } (3, 2)$$

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Triangle P is reflected in the line $x = 2$ to give triangle Q .

Triangle Q is reflected in the line $x = 5$ to give triangle R .

Describe fully the single transformation that maps P onto R .

(3 marks)

Follow the point $(1, 1)$

$$\text{Reflect in } x = 2: (1, 1) \rightarrow (3, 1)$$

$$\text{Reflect in } x = 5: (3, 1) \rightarrow (7, 1)$$

$$7 - 1 = 6$$

$$\text{Check: } 2 \times (5 - 2) = 6$$

$$\text{Translation by } (6, 0)$$

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Triangle A has vertices at $(2, 2)$, $(4, 2)$ and $(2, 6)$.

Triangle B has vertices at $(1, 1)$, $(2, 1)$ and $(1, 3)$.

Triangle C has vertices at $(-1, 1)$, $(-2, 1)$ and $(-1, 3)$.

a. Describe fully the single transformation that maps triangle A onto triangle B .

b. Describe fully the single transformation that maps triangle B onto triangle C .

(4 marks)

Every vertex of B is exactly half the matching vertex of A : $(2, 2)$ maps to $(1, 1)$, $(4, 2)$ maps to $(2, 1)$, $(2, 6)$ maps to $(1, 3)$

So A to B is an enlargement, scale factor $\frac{1}{2}$, centre the origin $(0, 0)$

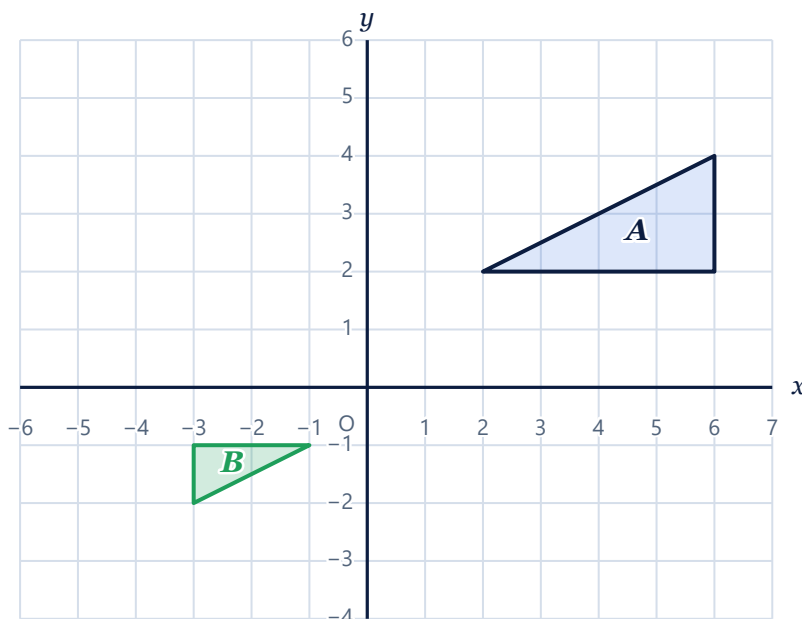
Every vertex of C has the same y -coordinate as the matching vertex of B with the sign of the x -coordinate changed

So B to C is a reflection in the y -axis (the line $x = 0$)

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Triangle A has vertices at (2, 2), (6, 2) and (6, 4).

- Triangle A is enlarged by scale factor $-\frac{1}{2}$, centre (0, 0), to give triangle B. Write down the coordinates of the vertices of triangle B.
- Describe fully the single transformation that maps triangle B back onto triangle A.



(3 marks)

a) Scale factor $-\frac{1}{2}$ with centre (0, 0) sends (x, y) to $(-\frac{x}{2}, -\frac{y}{2})$

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(2, 2) (-1, -1); (6, 2) (-3, -1); (6, 4) (-3, -2)

b) The inverse of an enlargement scale factor $-\frac{1}{2}$ about (0, 0) is an enlargement scale factor -2 about the same centre

Check

$(-1, -1) \times -2 = (2, 2)$. Correct

So the single transformation is an enlargement, scale factor -2, centre (0, 0)

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Triangle S has vertices $(1, 1)$, $(2, 1)$ and $(2, 3)$.

- a. S is reflected in the x -axis and the image is then enlarged by scale factor 2, centre $(0, 0)$. Work out the final position of each vertex. Then show that enlarging first and reflecting second gives the same final image.
- b. The point $(1, 1)$ is translated by the vector $(1, 0)$ and then enlarged by scale factor 2, centre $(0, 0)$. Show that enlarging first and translating second gives a different point, so the order can matter.

(4 marks)

a)

Reflect first, then enlarge

$$(1, 1) \rightarrow (1, -1) \rightarrow (2, -2)$$

$$(2, 1) \rightarrow (2, -1) \rightarrow (4, -2)$$

$$(2, 3) \rightarrow (2, -3) \rightarrow (4, -6)$$

Enlarge first, then reflect

$$(1, 1) \rightarrow (2, 2) \rightarrow (2, -2)$$

$$(2, 1) \rightarrow (4, 2) \rightarrow (4, -2)$$

$$(2, 3) \rightarrow (4, 6) \rightarrow (4, -6)$$

Same final image

b)

$$\text{Translate, then enlarge: } (1, 1) \rightarrow (2, 1) \rightarrow (4, 2)$$

$$\text{Enlarge, then translate: } (1, 1) \rightarrow (2, 2) \rightarrow (3, 2)$$

$$(4, 2) \neq (3, 2), \text{ so the order matters}$$

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A shape is rotated 180° about $(0, 0)$ and the image is then translated by the vector $(4, 2)$.

The two steps together are the same as a single rotation of 180° .

Find the coordinates of the centre of this single rotation.

(3 marks)

Follow a general point (x, y)

$$\text{Half turn about } (0, 0): (x, y) \rightarrow (-x, -y)$$

$$\text{Translate by } (4, 2): (-x, -y) \rightarrow (4 - x, 2 - y)$$

Centre = midpoint of start and end

$$x\text{-coordinate: } \frac{4}{2} = 2$$

$$y\text{-coordinate: } \frac{2}{2} = 1$$

$$\text{Check } (2, 1): (2, 1) \rightarrow (-2, -1) \rightarrow (2, 1)$$

$$\text{Centre } (2, 1)$$