Expanding three brackets

Worked solutions

GCSE Maths topic practice

This topic online

1 Expand and simplify $(x + 1)(x + 2)(x + 4)$ (2 marks)

Expand the first two brackets

$(x + 1)(x + 2) = x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

Multiply every term by $(x + 4)$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8$

$\boxed{x^3 + 7x^2 + 14x + 8}$

2 Expand and simplify $(x + 2)(x - 3)(x + 5)$ (3 marks)

Expand the first two brackets

$(x + 2)(x - 3) = x^2 - 3x + 2x - 6$

$= x^2 - x - 6$

Multiply every term by $(x + 5)$

$(x^2 - x - 6)(x + 5) = x^3 + 5x^2 - x^2 - 5x - 6x - 30$

$\boxed{x^3 + 4x^2 - 11x - 30}$

3 Expand and simplify $(x - 1)(x - 4)(x + 2)$ (3 marks)

Expand the first two brackets

$(x - 1)(x - 4) = x^2 - 4x - x + 4$

$= x^2 - 5x + 4$

Multiply every term by $(x + 2)$

$(x^2 - 5x + 4)(x + 2) = x^3 + 2x^2 - 5x^2 - 10x + 4x + 8$

$\boxed{x^3 - 3x^2 - 6x + 8}$

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4 Expand and simplify $(x + 1)^3$ (3 marks)

$(x + 1)^3$ means $(x + 1)(x + 1)(x + 1)$

$(x + 1)(x + 1) = x^2 + x + x + 1$

$= x^2 + 2x + 1$

Multiply every term by $(x + 1)$

$(x^2 + 2x + 1)(x + 1) = x^3 + x^2 + 2x^2 + 2x + x + 1$

$\boxed{x^3 + 3x^2 + 3x + 1}$

5 Expand and simplify $(x - 3)^2(x + 4)$ (3 marks)

Expand the squared bracket first

$(x - 3)^2 = (x - 3)(x - 3) = x^2 - 3x - 3x + 9$

$= x^2 - 6x + 9$

Multiply every term by $(x + 4)$

$(x^2 - 6x + 9)(x + 4) = x^3 + 4x^2 - 6x^2 - 24x + 9x + 36$

$\boxed{x^3 - 2x^2 - 15x + 36}$

6 Expand and simplify $(3x - 1)(x + 2)(x - 2)$ (3 marks)

Expand the last two brackets first (difference of two squares)

$(x + 2)(x - 2) = x^2 - 2x + 2x - 4$

$= x^2 - 4$

Multiply every term by $(3x - 1)$

$(3x - 1)(x^2 - 4) = 3x^3 - 12x - x^2 + 4$

$\boxed{3x^3 - x^2 - 12x + 4}$

7 Expand and simplify $(2x - 3)(x + 1)(x + 5)$ (3 marks)

Expand the last two brackets first

$(x + 1)(x + 5) = x^2 + 5x + x + 5$

$= x^2 + 6x + 5$

Multiply every term by $(2x - 3)$

$(2x - 3)(x^2 + 6x + 5) = 2x^3 + 12x^2 + 10x - 3x^2 - 18x - 15$

$\boxed{2x^3 + 9x^2 - 8x - 15}$

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8 A cuboid has length $(x + 3)$ cm, width $(x + 1)$ cm and height $(x - 2)$ cm, where $x > 2$.

The volume of the cuboid is V cm^3.

Show that $V = x^3 + 2x^2 - 5x - 6$ (3 marks)

Volume = length x width x height

$V = (x + 3)(x + 1)(x - 2)$

Expand the first two brackets

$(x + 3)(x + 1) = x^2 + x + 3x + 3$

$= x^2 + 4x + 3$

Multiply every term by $(x - 2)$

$V = (x^2 + 4x + 3)(x - 2) = x^3 - 2x^2 + 4x^2 - 8x + 3x - 6$

$\boxed{V = x^3 + 2x^2 - 5x - 6}$

9 $(x + 4)(x - 2)(x - 5) = x^3 + ax^2 + bx + c$, where a, b and c are integers.

a. Find the value of a.

b. Find the value of b.

c. Find the value of c. (4 marks)

Expand the first two brackets

$(x + 4)(x - 2) = x^2 - 2x + 4x - 8$

$= x^2 + 2x - 8$

Multiply every term by $(x - 5)$

$(x^2 + 2x - 8)(x - 5) = x^3 - 5x^2 + 2x^2 - 10x - 8x + 40$

$= x^3 - 3x^2 - 18x + 40$

Compare with $x^3 + ax^2 + bx + c$

a)

$\boxed{a = -3}$

b)

$\boxed{b = -18}$

c)

$\boxed{c = 40}$

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10 a. Expand and simplify $(2x - 1)(3x + 1)$

b. Hence expand and simplify $(x + 2)(2x - 1)(3x + 1)$

(4 marks)

a)

$(2x - 1)(3x + 1) = 6x^2 + 2x - 3x - 1$

$\boxed{6x^2 - x - 1}$

b)

Multiply every term of part a) by $(x + 2)$

$(x + 2)(6x^2 - x - 1) = 6x^3 - x^2 - x + 12x^2 - 2x - 2$

$\boxed{6x^3 + 11x^2 - 3x - 2}$

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