



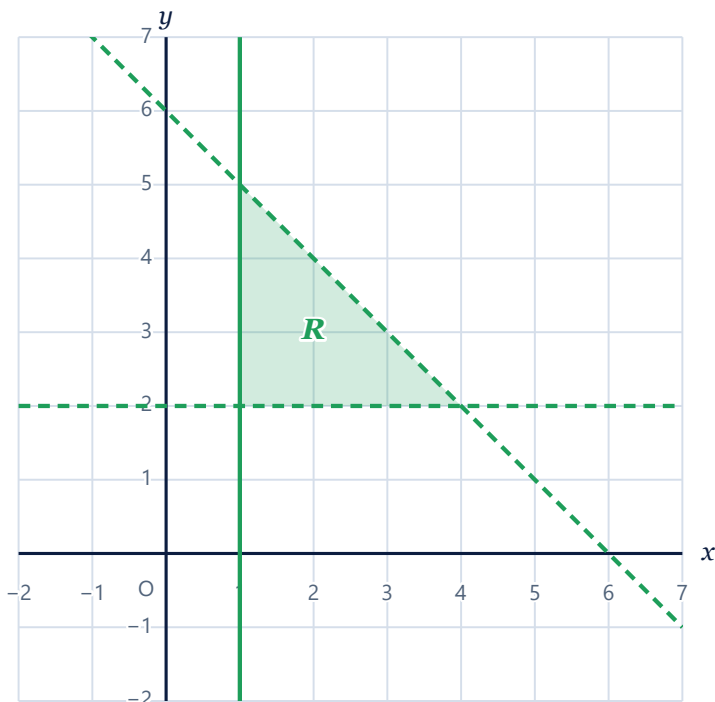
- 1 On the grid, shade the region that satisfies all three of these inequalities.

$$x \geq 1$$

$$y > 2$$

$$x + y < 6$$

Label the region R.



(3 marks)

Draw the line $x = 1$ (solid, because the inequality includes equals) and shade to the right of it

Draw the line $y = 2$ (dashed, because $y > 2$ is strict) and keep the region above it

Draw the line $x + y = 6$ (dashed) through $(0, 6)$ and $(6, 0)$ and keep the region below it

R is the triangle with vertices $(1, 2)$, $(4, 2)$ and $(1, 5)$

The stated corner coordinates are where the boundary lines meet. A dashed boundary is excluded from R, including any corner lying on that dashed line

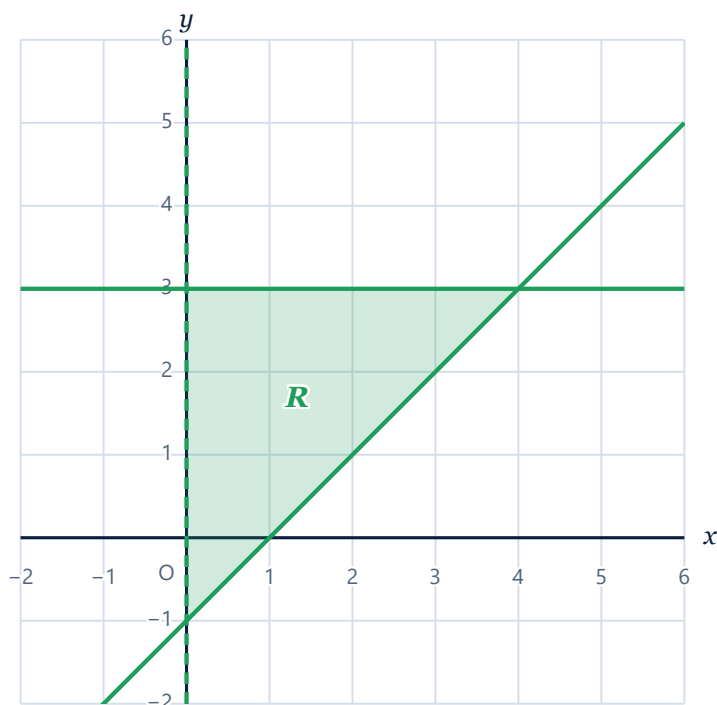
2 On the grid, shade the region that satisfies all three of these inequalities.

$$y \leq 3$$

$$y \geq x - 1$$

$$x > 0$$

Label the region R.



(3 marks)

Draw the horizontal line $y = 3$ (solid) and keep the region below it

Draw the line $y = x - 1$ (solid) through $(0, -1)$ and $(4, 3)$ and keep the region above it

Draw the line $x = 0$, the y-axis, dashed because $x > 0$ is strict, and keep the region to the right

R is the triangle with vertices $(0, -1)$, $(0, 3)$ and $(4, 3)$

The stated corner coordinates are where the boundary lines meet. A dashed boundary is excluded from R, including any corner lying on that dashed line

- 3 When a region is shaded on a graph, the boundary line for each inequality is drawn either solid or dashed.

Write "solid" or "dashed" for the boundary line of each inequality.

- a. $y > 1$
- b. $x \leq 5$
- c. $y \geq x + 2$
- d. $x + y < 6$

(2 marks)

$>$ or $<$ means dashed line

$\geq$ or $\leq$ means solid line

a)

dashed

b)

solid

c)

solid

d)

dashed

- 4 The region R satisfies all three of these inequalities.

$$x \geq 1$$

$$y \geq 2$$

$$x + y \leq 5$$

Find all the points in R whose coordinates are both integers.

(3 marks)

x must be an integer at least 1 and y an integer at least 2, with $x + y$ at most 5

If $x = 1$, then y can be 2, 3 or 4, giving (1, 2), (1, 3), (1, 4)

If $x = 2$, then y can be 2 or 3, giving (2, 2), (2, 3)

If $x = 3$, then y can only be 2, giving (3, 2). If $x = 4$ or more, $x + y$ would exceed 5

The six points are (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (3, 2)

5 The region R satisfies all three of these inequalities.

$$y > x$$

$$x \geq 0$$

$$y \leq 4$$

Amy says the point (2, 2) is in R.

Is Amy correct? You must explain your answer.

(3 marks)

Test (2, 2) in each inequality

$x \geq 0$ is satisfied since $2 \geq 0$, and $y \leq 4$ is satisfied since $2 \leq 4$

But $y > x$ requires $2 > 2$, which is false

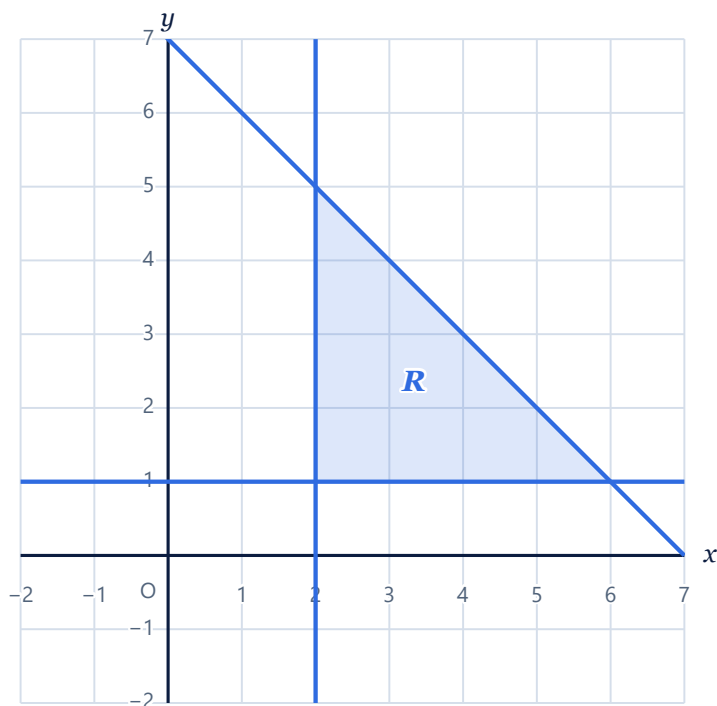
The point lies on the boundary line $y = x$, and a strict inequality does not include its boundary, so Amy is wrong

6 A region R is drawn on a coordinate grid.

R is the triangle with vertices (2, 1), (6, 1) and (2, 5), including its boundary.

The three sides of the triangle lie on the lines $x = 2$, $y = 1$ and $x + y = 7$

Find the three inequalities that define R.



(4 marks)

The region lies to the right of the vertical line $x = 2$, so $x \geq 2$

The region lies above the horizontal line $y = 1$, so $y \geq 1$

The region lies below the line $x + y = 7$. Check with a point inside, such as (3, 2): $3 + 2 = 5 \leq 7$, correct. So $x + y \leq 7$

All boundaries are included, so every inequality uses $\geq$ or $\leq$

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On the grid, shade the region that satisfies all four of these inequalities.

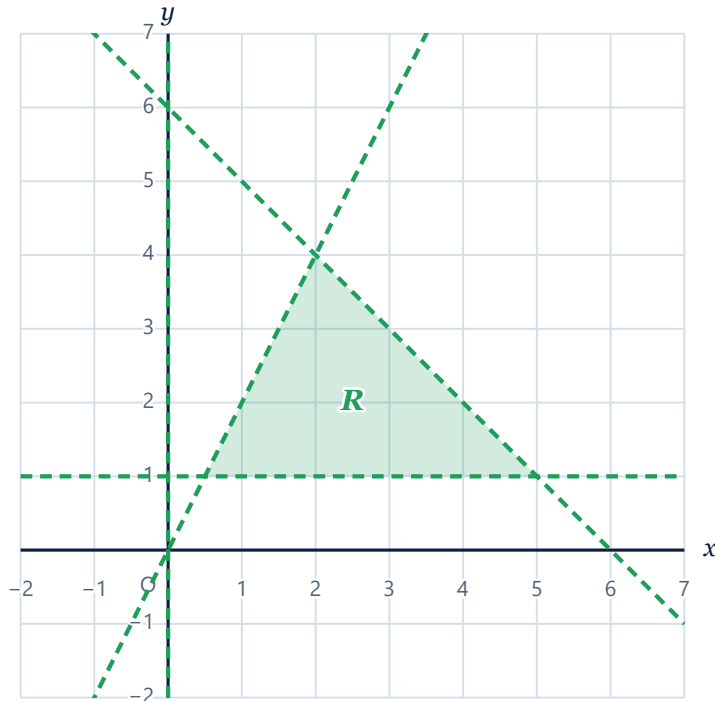
$$x > 0$$

$$y > 1$$

$$y < 2x$$

$$x + y < 6$$

Label the region R.



(4 marks)

Draw $y = 1$ (dashed) and keep the region above it

Draw $y = 2x$ (dashed) through $(0, 0)$, $(1, 2)$, $(2, 4)$ and keep the region below it, that is to the right of the line

Draw $x + y = 6$ (dashed) through $(0, 6)$ and $(6, 0)$ and keep the region below it

$x > 0$ holds automatically in this triangle, since $y < 2x$ with $y > 1$ forces $x > \frac{1}{2}$

R is the triangle with vertices $(\frac{1}{2}, 1)$, $(5, 1)$ and $(2, 4)$, with every boundary dashed

The stated corner coordinates are where the boundary lines meet. A dashed boundary is excluded from R, including any corner lying on that dashed line

- 8 The region R satisfies all three of these inequalities.

$$x > 0$$

$$y \geq 1$$

$$x + y < 5$$

Write down the coordinates of all the points in R with whole number coordinates.

(3 marks)

$x > 0$ and $y \geq 1$ so x, y are 1, 2, 3, ...

$x + y < 5$ so $x + y \leq 4$

$x = 1$: $y = 1, 2, 3$

$x = 2$: $y = 1, 2$

$x = 3$: $y = 1$

$(1,1) (1,2) (1,3) (2,1) (2,2) (3,1)$

- 9 The region R satisfies all three of these inequalities.

$$x + 2y \leq 10$$

$$x \geq 0$$

$$y > 1$$

Which of these points are in R?

P(4, 3) Q(6, 2) S(2, 1) T(0, 5)

(3 marks)

P(4, 3)

$4 + 6 = 10 \leq 10, 4 \geq 0, 3 > 1$ - in R

Q(6, 2)

$6 + 4 = 10 \leq 10, 6 \geq 0, 2 > 1$ - in R

S(2, 1)

$1 > 1$ is false - not in R

T(0, 5)

$0 + 10 = 10 \leq 10, 0 \geq 0, 5 > 1$ - in R

P, Q and T

- 10 The region R is bounded by three lines: $x = 1$, $y = 2$ and $x + y = 8$.

R is to the right of $x = 1$, above $y = 2$ and below $x + y = 8$.

The lines $x = 1$ and $x + y = 8$ are solid. The line $y = 2$ is dashed.

Write down the three inequalities that define R.

(3 marks)

right of $x = 1$, solid: $x \geq 1$

above $y = 2$, dashed: $y > 2$

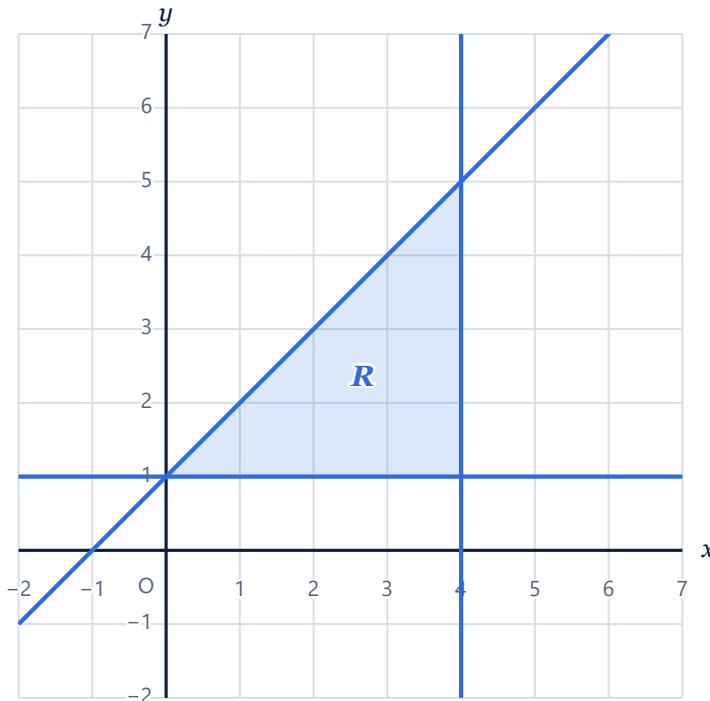
below $x + y = 8$, solid: $x + y \leq 8$

$$x \geq 1, y > 2, x + y \leq 8$$

- 11 A region R is drawn on a coordinate grid.

R is the triangle with vertices $(0, 1)$, $(4, 1)$ and $(4, 5)$, including its boundary.

Find the three inequalities that define R.



(4 marks)

The side from $(0, 1)$ to $(4, 1)$ lies on $y = 1$, and the region is above it, so $y \geq 1$

The side from $(4, 1)$ to $(4, 5)$ lies on $x = 4$, and the region is to the left, so $x \leq 4$

The side from $(0, 1)$ to $(4, 5)$ has gradient $(5 - 1) \div (4 - 0) = 1$ and y -intercept 1, so it lies on $y = x + 1$

Check with a point inside, such as $(3, 2)$: $2 \leq 3 + 1$, correct, so $y \leq x + 1$

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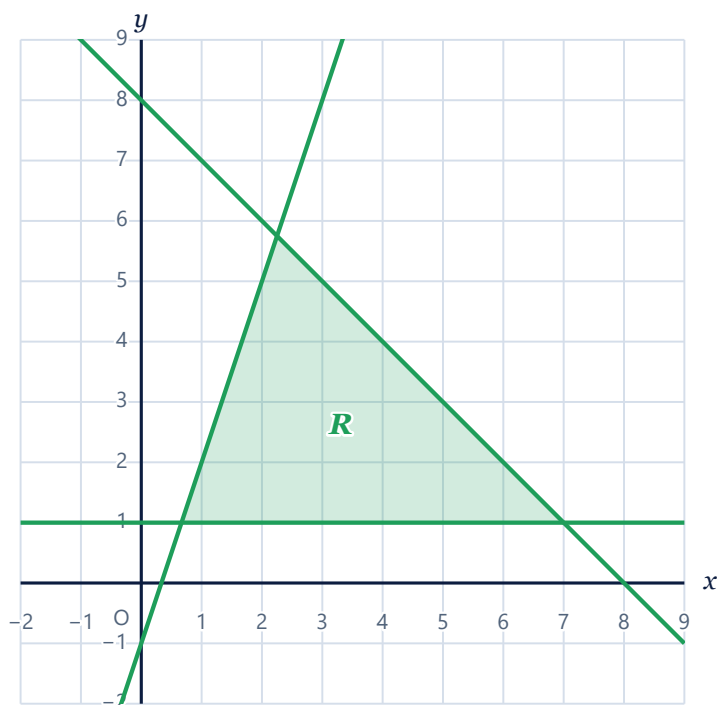
a. On the grid, shade the region R that satisfies all three of these inequalities.

$$y \geq 1$$

$$x + y \leq 8$$

$$y \leq 3x - 1$$

b. Priya says the point (3, 8) is in R. Explain why Priya is wrong.



(5 marks)

Draw $y = 1$ (solid) and keep the region above it

Draw $x + y = 8$ (solid) through (0, 8) and (8, 0) and keep the region below it

Draw $y = 3x - 1$ (solid) through (0, -1), (1, 2), (2, 5) and keep the region below it, that is to the right of the line

The vertices of R are where the lines cross

$y = 1$ meets $y = 3x - 1$ at $(\frac{2}{3}, 1)$; $y = 1$ meets $x + y = 8$ at (7, 1);

$y = 3x - 1$ meets $x + y = 8$ where $x + 3x - 1 = 8$, giving $(\frac{9}{4}, \frac{23}{4})$

For part b, test (3, 8) in $x + y \leq 8$

$3 + 8 = 11$, which is greater than 8, so the point is not in R

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A straight line passes through (0, 1) and (4, 3).

The region R is on or above this line, on or below the line $y = 3$, and on or to the right of the y -axis.

Write down the three inequalities that define R.

(4 marks)

$$\text{gradient} = \{3 - 1\} / \{4 - 0\} = \frac{1}{2}$$

$$y\text{-intercept} = 1, \text{ line is } y = \frac{1}{2}x + 1$$

$$\text{on or above: } y \geq \frac{1}{2}x + 1$$

$$\text{on or below } y = 3: y \leq 3$$

$$\text{on or right of the } y\text{-axis: } x \geq 0$$

$$y \geq \frac{1}{2}x + 1, y \leq 3, x \geq 0$$

14 The region R satisfies all four of these inequalities.

$$x \geq 0$$

$$y \geq 0$$

$$2x + y \leq 10$$

$$y \leq x + 2$$

The point (x, y) is in R and both coordinates are integers.

Find the greatest possible value of $x + y$

(5 marks)

To make $x + y$ large, take points near where $2x + y = 10$ meets $y = x + 2$, which is at $x = \frac{8}{3}$, $y = \frac{14}{3}$, not an integer point

Try integer x -values near $\frac{8}{3}$. If $x = 3$: $2x + y \leq 10$ gives $y \leq 4$, and $y \leq x + 2$ gives $y \leq 5$, so the best is $y = 4$ and $x + y = 7$

If $x = 2$: $y \leq 6$ and $y \leq 4$, so the best is $y = 4$ and $x + y = 6$

If $x = 4$: $y \leq 2$ and $y \leq 6$, so the best is $y = 2$ and $x + y = 6$

To check every possibility

$x \geq 0$ and $2x + y \leq 10$ with $y \geq 0$ give integer $x = 0, 1, 2, 3, 4$

or 5. Their

greatest

allowed

integer y -

values are 2,

3, 4, 4, 2 and

0

respectively.

The

corresponding

totals $x + y$

are 2, 4, 6,

7, 6 and 5,

so no point

has a larger

total

The greatest possible value is 7, at the point (3, 4)

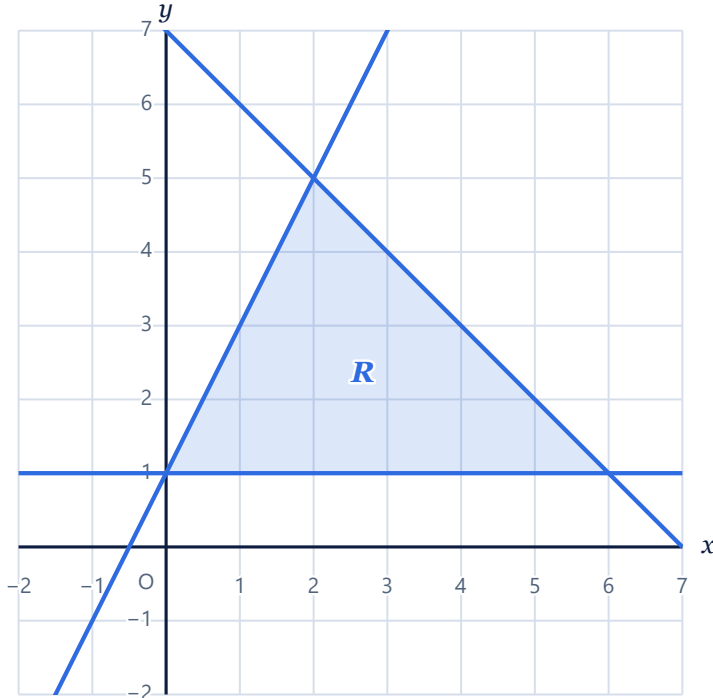
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A region R is drawn on a coordinate grid.

R is the triangle with vertices (0, 1), (6, 1) and (2, 5), including its boundary.

Find the three inequalities that define R.



(5 marks)

The side from (0, 1) to (6, 1) lies on $y = 1$, and the region is above it, so $y \geq 1$

The side from (0, 1) to (2, 5) has gradient $(5 - 1) \div (2 - 0) = 2$ and y-intercept 1, so it lies on $y = 2x + 1$. The region is below this line, check with (2, 2): $2 \leq 5$, correct, so $y \leq 2x + 1$

The side from (6, 1) to (2, 5) has gradient $(5 - 1) \div (2 - 6) = -1$, so it lies on $y = -x + 7$, that is $x + y = 7$. The region is below it, check with (2, 2): $2 + 2 = 4 \leq 7$, correct, so $x + y \leq 7$

All boundaries are included, so every inequality uses $\geq$ or $\leq$

16 The region R satisfies all three of these inequalities.

$$x \geq 1$$

$$y > 2$$

$$x + y \leq 8$$

- Work out how many points in R have whole number coordinates.
- Work out the greatest value of $2x + y$ for points in R with whole number coordinates.

(5 marks)

a)

$y > 2$ so $y \geq 3$ for whole numbers

$$x = 1: y = 3, 4, 5, 6, 7 \text{ (5)}$$

$$x = 2: y = 3, 4, 5, 6 \text{ (4)}$$

$$x = 3: y = 3, 4, 5 \text{ (3)}$$

$$x = 4: y = 3, 4 \text{ (2)}$$

$$x = 5: y = 3 \text{ (1)}$$

$$5 + 4 + 3 + 2 + 1 = 15$$

15

b)

check the corner points of R

$$(5, 3): 2 \times 5 + 3 = 13$$

$$(4, 4): 2 \times 4 + 4 = 12$$

$$(1, 7): 2 \times 1 + 7 = 9$$

13