



1 y is directly proportional to x . $y = 24$ when $x = 8$.

a. Find y when $x = 5$.

b. Find x when $y = 45$.

(3 marks)

$$\begin{array}{rcl}
 y & = & kx \\
 24 & = & k \times 8 \\
 \div 8 & & \div 8 \\
 k & = & 3 \\
 y & = & 3x \\
 \text{a)} & & \\
 y & = & 3 \times 5 \\
 \boxed{15} & & \\
 \text{b)} & & \\
 45 & = & 3x \\
 \div 3 & & \div 3 \\
 x & = & 15 \\
 \boxed{15} & &
 \end{array}$$

2

The cost, C pounds, of a square tile is directly proportional to the square of the length of its side, s cm.

A tile with side 10 cm costs £12.

a. Show that $C = 0.12s^2$.

b. Work out the cost of a tile with side 15 cm.

(4 marks)

a.)

$$C = ks^2$$

$$12 = k \times 10^2$$

$$12 = 100k$$

$$\div 100 \quad k = 0.12 \quad \div 100$$

$$C = 0.12s^2$$

b.)

$$C = 0.12 \times 15^2$$

$$= 0.12 \times 225$$

$$£27$$

3

The time, T hours, taken to paint a fence is inversely proportional to the number of painters, n .

4 painters take 9 hours.

a. Write down a formula for T in terms of n .

b. How long would 6 painters take?

c. How many painters are needed to paint the fence in 2.4 hours?

(4 marks)

a)

$$\begin{aligned}
 T &= \frac{k}{n} \\
 9 &= \frac{k}{4} \\
 \times 4 \quad k &= 36 \quad \times 4 \\
 \boxed{T = \frac{36}{n}}
 \end{aligned}$$

b)

$$\begin{aligned}
 T &= \frac{36}{6} \\
 \boxed{6 \text{ hours}}
 \end{aligned}$$

c)

$$\begin{aligned}
 2.4 &= \frac{36}{n} \\
 n &= \frac{36}{2.4} \\
 \boxed{15 \text{ painters}}
 \end{aligned}$$

4

y is directly proportional to the square of x .

When $x = 4$, $y = 48$.

Find the value of y when $x = 5$.

(3 marks)

Directly proportional to x^2 means that $y \div x^2$ is a fixed number, called the constant of proportionality k . So write $y = kx^2$

Substitute

$$48 = k \times 16, \text{ so } k = 3 \text{ and } y = 3x^2$$

$$\text{When } x = 5, y = 3 \times 25 = 75$$

$$\boxed{y = 75}$$

5 p is inversely proportional to q .

When $q = 5$, $p = 12$.

- Find a formula for p in terms of q .
- Find the value of p when $q = 8$.

(3 marks)

Inversely proportional means that the product pq is constant

$$pq = k. \text{ Dividing by } q \text{ gives } p = k/q$$

Substitute

$$12 = \frac{k}{5}, \text{ so } k = 60 \text{ and } p = \frac{60}{q}$$

$$\text{When } q = 8, p = 60 \div 8 = 7.5$$

6 y is inversely proportional to x .

When $x = 4$, $y = 9$.

- Find the value of y when $x = 6$.
- Find the value of y when $x = 12$.

(3 marks)

$$y = \frac{k}{x} \text{ with } k = 4 \times 9 = 36$$

$$\text{a) } y = 36 \div 6 = 6$$

$$\text{b) } y = 36 \div 12 = 3$$

Check

$$6 \times 6 = 36 \text{ and } 12 \times 3 = 36, \text{ so } k \text{ is constant both ways}$$

7 y is inversely proportional to the square of x .

When $x = 3$, $y = 4$.

Find the negative value of x when $y = 9$.

(3 marks)

$$\text{Write } y = \frac{k}{x^2}$$

Substitute

$$4 = \frac{k}{9}, \text{ so } k = 36 \text{ and } y = \frac{36}{x^2}$$

$$\text{When } y = 9: 9 = \frac{36}{x^2}, \text{ so } x^2 = 4$$

$$x = 2 \text{ or } x = -2, \text{ so the negative value is } x = -2$$

$$\boxed{x = -2}$$

8

The force F newtons between two magnets is inversely proportional to the square of the distance d cm between them.

When $d = 2$, $F = 60$.

- Find a formula for F in terms of d .
- Find the positive value of d when $F = 15$.

(4 marks)

$$\text{Write } F = \frac{k}{d^2}$$

Substitute

$$60 = \frac{k}{4}, \text{ so } k = 240 \text{ and } F = \frac{240}{d^2}$$

$$\text{When } F = 15: 15 = \frac{240}{d^2}, \text{ so } d^2 = 16$$

The positive value is $d = 4$

9

y is inversely proportional to the cube of x . $y = 4$ when $x = 2$.

- Show that $y = \frac{1}{2}$ when $x = 4$.
- Find the value of x when $y = \frac{32}{125}$.

(4 marks)

$$\begin{array}{l} y = \frac{k}{x^3} \\ 4 = \frac{k}{2^3} = \frac{k}{8} \\ \times 8 \quad k = 32 \quad \times 8 \\ y = \frac{32}{x^3} \end{array}$$

a)

$$y = \frac{32}{4^3} = \frac{32}{64}$$

$$\boxed{y = \frac{1}{2}}$$

b)

$$\begin{array}{l} \frac{32}{125} = \frac{32}{x^3} \\ x^3 = 125 \\ \sqrt[3]{\quad} \quad x = 5 \quad \sqrt[3]{\quad} \end{array}$$

$$\boxed{x = 5}$$

- 10 x and y are positive quantities.
 y is inversely proportional to the square of x .
 x is increased by 25%.
Work out the percentage change in y .

(4 marks)

$$\text{Write } y = \frac{k}{x^2}$$

$$\text{The new value of } x \text{ is } 1.25x, \text{ so the new } y \text{ is } \frac{k}{(1.25x)^2} = \frac{k}{1.5625x^2}$$

$$1 \div 1.5625 = 0.64, \text{ so the new } y \text{ is } 0.64 \text{ times the old } y$$

This is a 36% decrease

36% decrease

- 11 y is inversely proportional to x .
When $x = 3, y = 20$.

Work out the value of y when $x = 5$.

(2 marks)

$$k = 3 \times 20 = 60, \text{ so } y = \frac{60}{x}$$

$$\text{When } x = 5, y = 60 \div 5 = 12$$

12

- 12 The distance d metres a dropped object falls is directly proportional to the square of the time t seconds.
When $t = 3, d = 45$.

Work out the value of d when $t = 6$.

(2 marks)

$$d = kt^2 \text{ with } 45 = k \times 9, \text{ so } k = 5$$

$$\text{When } t = 6, d = 5 \times 36 = 180$$

Check

$$\text{doubling } t \text{ multiplies } d \text{ by } 4, \text{ and } 45 \times 4 = 180$$

180

13 y is directly proportional to the square of x .

x is increased by 20%.

Show that y increases by 44%.

Assume x and y are positive.

(3 marks)

$$y = kx^2$$

The new value of x is $1.2x$

$$\text{New } y = k(1.2x)^2 = 1.44kx^2$$

1.44 times the original y is a 44% increase