



1 $\mathbf{a} = (3, 5)$ and $\mathbf{b} = (2, -1)$.

Work out $\mathbf{a} + \mathbf{b}$ as a column vector.

(2 marks)

Add the top numbers

$$3 + 2 = 5$$

Add the bottom numbers

$$5 + (-1) = 4$$

$$\text{So } \mathbf{a} + \mathbf{b} = (5, 4)$$

$$\boxed{(5, 4)}$$

2 $\mathbf{a} = (4, -3)$ and $\mathbf{b} = (1, 6)$.

Work out $2\mathbf{a} + \mathbf{b}$ as a column vector.

(2 marks)

$$2\mathbf{a} = (2 \times 4, 2 \times -3) = (8, -6)$$

$$2\mathbf{a} + \mathbf{b} = (8 + 1, -6 + 6) = (9, 0)$$

$$\boxed{(9, 0)}$$

3 $\mathbf{p} = (5, 2)$ and $\mathbf{q} = (3, -4)$.

Work out $\mathbf{p} - 2\mathbf{q}$ as a column vector.

(2 marks)

$$2\mathbf{q} = (6, -8)$$

$$\mathbf{p} - 2\mathbf{q} = (5 - 6, 2 - (-8)) = (-1, 10)$$

$$\boxed{(-1, 10)}$$

4 $\mathbf{a} = (-2, 5)$ and $\mathbf{b} = (3, -1)$.

Work out $3\mathbf{a} - 2\mathbf{b}$ as a column vector.

(2 marks)

$$3\mathbf{a} = (-6, 15) \text{ and } 2\mathbf{b} = (6, -2)$$

$$3\mathbf{a} - 2\mathbf{b} = (-6 - 6, 15 - (-2)) = (-12, 17)$$

$$\boxed{(-12, 17)}$$

- 5 OAB is a triangle. $OA = \mathbf{a}$ and $OB = \mathbf{b}$.
Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector AB .

(2 marks)

$$\begin{aligned} \text{A to B: go via O} \\ AB &= AO + OB \\ &= -\mathbf{a} + \mathbf{b} \end{aligned}$$

$$\boxed{AB = -\mathbf{a} + \mathbf{b}}$$

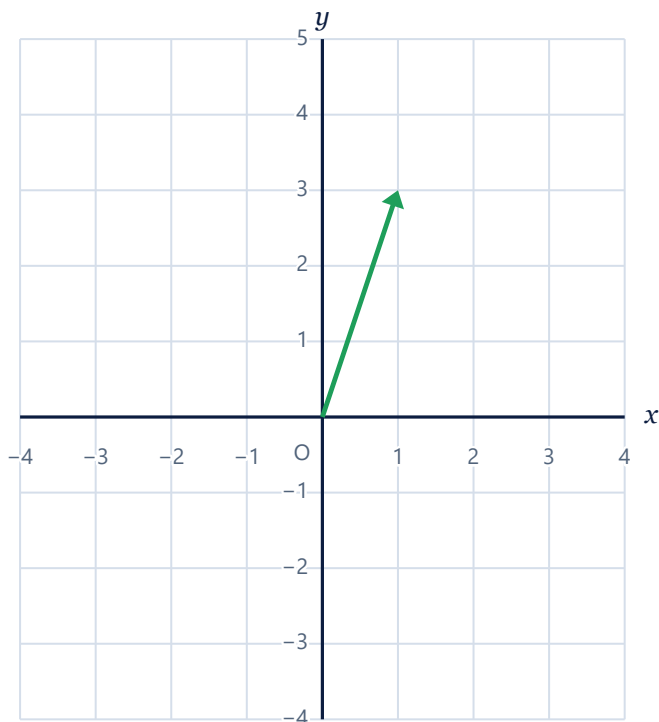
- 6 P is the point (3, 4) and Q is the point (-1, 6).
Write PQ as a column vector.

(2 marks)

$$\begin{aligned} PQ &= Q - P \\ \text{across: } -1 - 3 &= -4 \\ \text{up: } 6 - 4 &= 2 \end{aligned}$$

$$\boxed{PQ = \begin{pmatrix} -4 \\ 2 \end{pmatrix}}$$

7

 $\mathbf{a} = (3, -1)$ and $\mathbf{b} = (-1, 2)$.On the grid, draw and label the vector $\mathbf{a} + 2\mathbf{b}$.

(3 marks)

$$2\mathbf{b} = (-2, 4)$$

$$\mathbf{a} + 2\mathbf{b} = (3 - 2, -1 + 4) = (1, 3)$$

Draw an arrow going 1 square right and 3 squares up, with the arrowhead at the finishing end

$(1, 3)$

8

$$k(2, -3) + (5, 1) = (13, m)$$

where k and m are numbers.Find the value of k and the value of m .

(3 marks)

Top numbers

$$2k + 5 = 13, \text{ so } 2k = 8 \text{ and } k = 4$$

Bottom numbers

$$k \times (-3) + 1 = m$$

$$m = 4 \times (-3) + 1 = -12 + 1 = -11$$

9 OABC is a parallelogram. $OA = \mathbf{a}$ and $OC = \mathbf{c}$.

Write down, in terms of $\mathbf{a}$ and $\mathbf{c}$, the vectors

a. OB

b. AC

c. BA

(3 marks)

a)

$$AB = OC = \mathbf{c}$$

$$OB = OA + AB$$

$$\boxed{\mathbf{a} + \mathbf{c}}$$

b)

$$AC = AO + OC$$

$$\boxed{-\mathbf{a} + \mathbf{c}}$$

c)

BA is the reverse of $AB = \mathbf{c}$

$$\boxed{-\mathbf{c}}$$

10 Work out the magnitude (length) of each of these vectors.

a. (5, 12)

b. (4, -7). Give your answer correct to 3 significant figures.

(3 marks)

a)

$$\begin{aligned}\sqrt{5^2 + 12^2} &= \sqrt{25 + 144} \\ &= \sqrt{169}\end{aligned}$$

$$\boxed{13}$$

b)

$$\begin{aligned}\sqrt{4^2 + (-7)^2} &= \sqrt{16 + 49} \\ &= \sqrt{65}\end{aligned}$$

$$\boxed{8.06}$$

11 $\mathbf{p} = (6, -9)$ and $\mathbf{q} = (-2, 3)$.

Show that $\mathbf{p}$ and $\mathbf{q}$ are parallel.

(2 marks)

$$-2 \times (-3) = 6$$

$$3 \times (-3) = -9$$

$$\text{so } \mathbf{p} = -3\mathbf{q}$$

$\mathbf{p}$ and $\mathbf{q}$ are parallel

12 A is the point (1, 2) and B is the point (5, 8).

ABC is a straight line and $BC = \frac{1}{2}AB$.

Work out the coordinates of C.

The points are in the order A, B, C, so C lies beyond B.

(3 marks)

$$\text{vector } AB = (5 - 1, 8 - 2) = (4, 6)$$

$$BC = \frac{1}{2}AB = (2, 3)$$

$$C = B + BC = (5 + 2, 8 + 3) = (7, 11)$$

(7, 11)

13 $m(2, 1) + n(1, 3) = (4, 7)$

where m and n are numbers.

Find the value of m and the value of n .

(3 marks)

Top numbers

$$2m + n = 4$$

Bottom numbers

$$m + 3n = 7$$

From the first equation $n = 4 - 2m$. Substitute: $m + 3(4 - 2m) = 7$, so $m + 12 - 6m = 7$
and $-5m = -5$, giving $m = 1$

$$\text{Then } n = 4 - 2 = 2$$

Check

$$1 + 3 \times 2 = 7. \text{ Correct}$$

- 14 The vector $(4, k)$ is parallel to the vector $(6, 9)$.

Work out the value of k .

(2 marks)

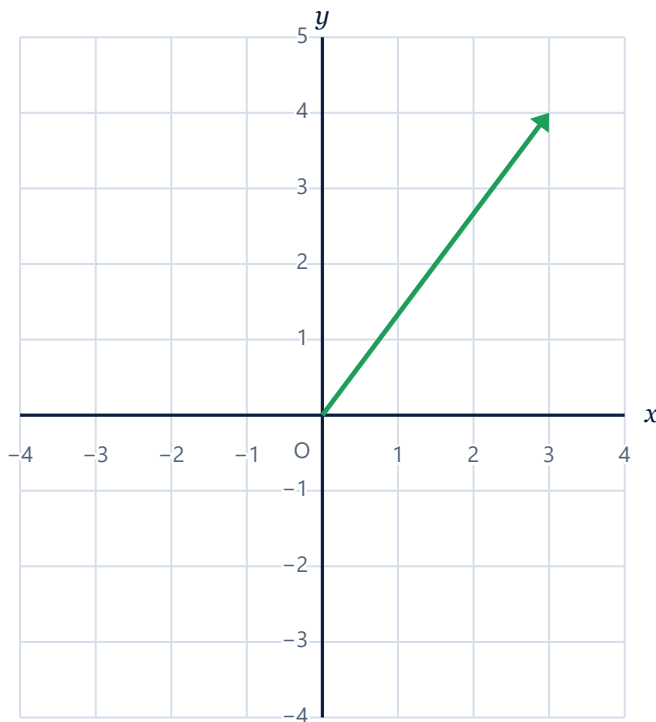
$$4 \div 6 = \frac{2}{3}$$

$$k = \frac{2}{3} \times 9$$

$$k = 6$$

- 15 $\mathbf{a} = (2, 1)$ and $\mathbf{b} = (1, -2)$.

On the grid, draw and label the vector $2\mathbf{a} - \mathbf{b}$.



(3 marks)

$$2\mathbf{a} = (4, 2)$$

$$2\mathbf{a} - \mathbf{b} = (4 - 1, 2 - (-2)) = (3, 4)$$

Draw an arrow going 3 squares right and 4 squares up, with an arrowhead

$$(3, 4)$$

16

 $\mathbf{a} = (1, 2)$ and $\mathbf{b} = (3, -1)$. $(9, 4) = m\mathbf{a} + n\mathbf{b}$, where m and n are numbers.Find the value of m and the value of n .

(4 marks)

$$\text{top: } m + 3n = 9$$

$$\text{bottom: } 2m - n = 4$$

$$\begin{array}{rcl}
 n & = & 2m - 4 \\
 m + 3(2m - 4) & = & 9 \\
 7m - 12 & = & 9 \\
 +12 & & +12 \\
 7m & = & 21 \\
 \div 7 & & \div 7 \\
 m & = & 3
 \end{array}$$

$$n = 2 \times 3 - 4 = 2$$

$$m = 3, n = 2$$

17

 $AB = 2\mathbf{a} + 4\mathbf{b}$ and $BC = 3\mathbf{a} + 6\mathbf{b}$.

Show that A, B and C lie on a straight line.

(3 marks)

$$\begin{aligned}
 BC &= 3\mathbf{a} + 6\mathbf{b} \\
 &= \frac{3}{2}(2\mathbf{a} + 4\mathbf{b}) \\
 &= \frac{3}{2}AB
 \end{aligned}$$

BC is parallel to AB

both lines contain the point B

$$A, B \text{ and } C \text{ are on a straight line}$$

18

OAB is a triangle. $OA = \mathbf{a}$ and $OB = \mathbf{b}$.

P is the point on AB such that $AP : PB = 2 : 1$.

Express OP in terms of $\mathbf{a}$ and $\mathbf{b}$. Give your answer in its simplest form.

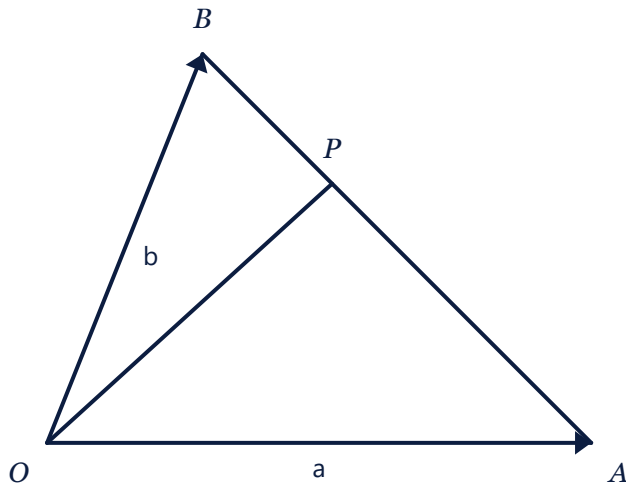


Diagram NOT accurately drawn

(4 marks)

$$AB = OB - OA = b - a$$

AP

$$PB = 2 : 1, \text{ so } AP = \left(\frac{2}{3}\right)AB = \left(\frac{2}{3}\right)(b - a)$$

$$OP = OA + AP = a + \left(\frac{2}{3}\right)(b - a)$$

$$OP = a - \left(\frac{2}{3}\right)a + \left(\frac{2}{3}\right)b = \left(\frac{1}{3}\right)a + \left(\frac{2}{3}\right)b$$

$$\boxed{\left(\frac{1}{3}\right)a + \left(\frac{2}{3}\right)b}$$

19

OABC is a trapezium. $OA = \mathbf{a}$ and $OC = \mathbf{c}$.

CB is parallel to OA and $CB = 2\mathbf{a}$.

M is the midpoint of AB.

Find OM in terms of $\mathbf{a}$ and $\mathbf{c}$. Give your answer in its simplest form.

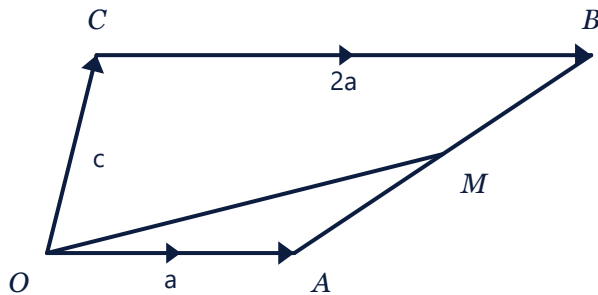


Diagram NOT accurately drawn

(4 marks)

$$OB = OC + CB = c + 2a$$

$$AB = OB - OA = c + 2a - a = a + c$$

$$AM = \frac{1}{2}AB = \frac{1}{2}(a + c)$$

$$OM = OA + AM = a + \frac{1}{2}a + \frac{1}{2}c = \left(\frac{3}{2}\right)a + \left(\frac{1}{2}\right)c$$

$$\boxed{\left(\frac{3}{2}\right)a + \left(\frac{1}{2}\right)c}$$