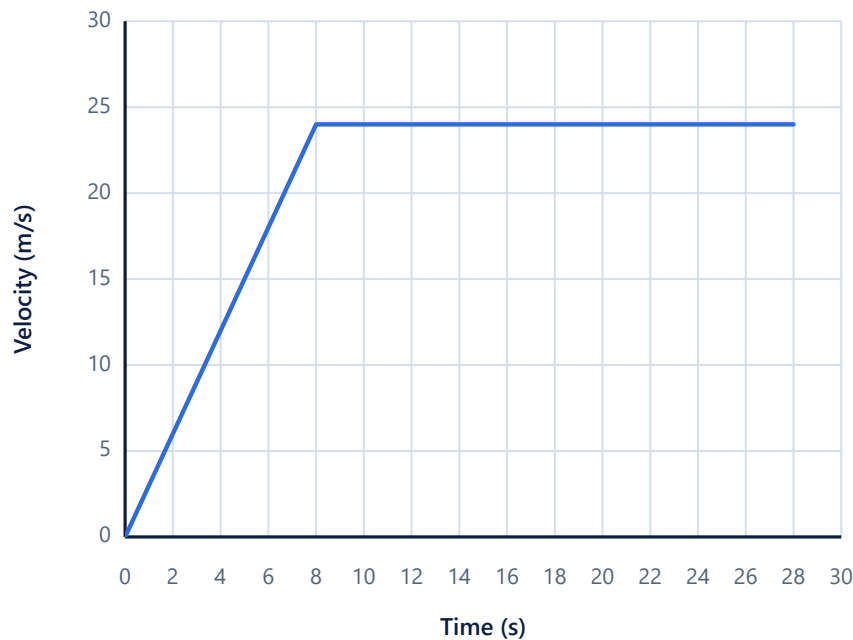




- 1** A car starts from rest and accelerates uniformly to a velocity of 24 m/s in 8 seconds. It then travels at a constant velocity of 24 m/s for a further 20 seconds.
- Work out the acceleration of the car during the first 8 seconds.
 - Work out the total distance travelled by the car in the 28 seconds.



(3 marks)

a) Acceleration = change in velocity divided by time = $\frac{24}{8} = 3$
m/s²

b) Distance = area under the velocity-time graph

First 8 seconds (triangle)

$$\frac{1}{2} \times 8 \times 24 = 96 \text{ m}$$

Next 20 seconds (rectangle)

$$20 \times 24 = 480 \text{ m}$$

$$\text{Total distance} = 96 + 480 = 576 \text{ m}$$

2

A velocity-time graph shows a cyclist who accelerates uniformly from rest to 12 m/s in 8 seconds, then travels at a constant 12 m/s for 10 seconds.

- Work out the acceleration during the first 8 seconds.
- Work out the total distance travelled in the 18 seconds.

(3 marks)

a) gradient

$$\frac{12}{8} = 1.5$$

$$\boxed{1.5 \text{ m/s}^2}$$

b) Area under the graph

$$\text{triangle} = \frac{1}{2} \times 8 \times 12 = 48$$

$$\text{rectangle} = 12 \times 10 = 120$$

$$48 + 120 = 168$$

$$\boxed{168 \text{ m}}$$

3

A runner's velocity-time graph is a smooth curve. The table gives her velocity v m/s at time t seconds.

t : 0, 10, 20, 30 v : 0, 4.2, 6.6, 7.8

Use the trapezium rule with 3 strips of equal width to estimate the distance the runner travels in the first 30 seconds.

(3 marks)

Distance is the area under the velocity-time graph. Use 3 trapezia of width 10

First strip

$$\frac{1}{2} \times 10 \times (0 + 4.2) = 21$$

Second strip

$$\frac{1}{2} \times 10 \times (4.2 + 6.6) = 54$$

Third strip

$$\frac{1}{2} \times 10 \times (6.6 + 7.8) = 72$$

$$\text{Estimate} = 21 + 54 + 72 = 147 \text{ m}$$

$$\boxed{147 \text{ m}}$$

4

A train travels at a constant velocity of 20 m/s for 30 seconds. It then brakes uniformly and comes to rest after a further 8 seconds.

- Work out the deceleration of the train while it is braking.
- Work out the total distance travelled by the train.

(4 marks)

a) velocity falls 20 in 8 seconds

$$\frac{20}{8} = 2.5$$

$$2.5 \text{ m/s}^2 \text{ deceleration}$$

(gradient = -2.5)

b) Area under the graph

$$20 \times 30 = 600$$

$$\frac{1}{2} \times 8 \times 20 = 80$$

$$600 + 80 = 680$$

$$680 \text{ m}$$

5

A cyclist accelerates away from traffic lights. Her speed-time graph is a smooth curve that gets less steep as time increases. The table gives her speed v m/s at time t seconds.

t : 0, 5, 10, 15, 20 v : 0, 3.6, 5.4, 6.4, 7.0

- Use the trapezium rule with 4 strips of equal width to estimate the distance travelled in the first 20 seconds.
- State, with a reason, whether your answer to part (a) is an underestimate or an overestimate of the actual distance.

(4 marks)

$$\text{a) Area} = \frac{1}{2} \times 5 \times (0 + 3.6) + \frac{1}{2} \times 5 \times (3.6 + 5.4) + \frac{1}{2} \times 5 \times (5.4 + 6.4) + \frac{1}{2} \times 5 \times (6.4 + 7.0)$$

$$= 9 + 22.5 + 29.5 + 33.5 = 94.5 \text{ m}$$

$$\text{(Equivalently } \frac{1}{2} \times 5 \times (0 + 2 \times 3.6 + 2 \times 5.4 + 2 \times 6.4 + 7.0) = 2.5 \times 37.8 = 94.5.)$$

b) The graph is a curve that gets less steep (concave), so each chord lies below the curve and each trapezium misses some of the area. The estimate is an underestimate

6

A cyclist's velocity increases uniformly from 4 m/s to 10 m/s over 12 seconds.

- Work out the acceleration of the cyclist.
- Work out the distance travelled in the 12 seconds.

(4 marks)

a)

$$\frac{10 - 4}{12} = 0.5$$

$$\boxed{0.5 \text{ m/s}^2}$$

b) Trapezium area

$$= \frac{1}{2} \times (4 + 10) \times 12$$

$$= 7 \times 12$$

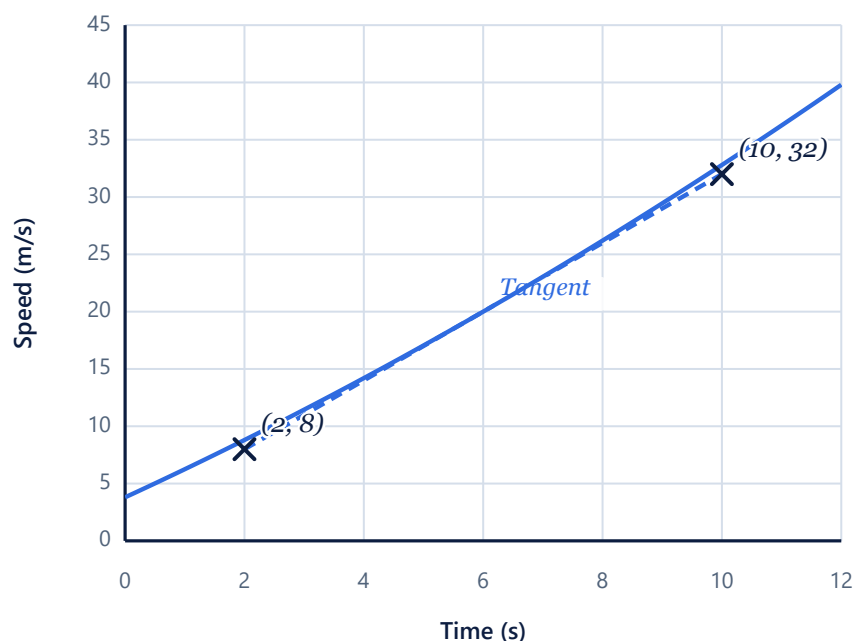
$$\boxed{84 \text{ m}}$$

7

The speed-time graph for a car is a smooth curve for $0 \leq t \leq 12$, where t is the time in seconds and the speed is in m/s.

A tangent drawn by eye to the curve at $t = 6$ passes through the points $(2, 8)$ and $(10, 32)$.

- Use the tangent to estimate the acceleration of the car at $t = 6$.
- Explain why your answer is only an estimate.



(4 marks)

a) Acceleration at $t = 6$ is the gradient of the tangent at $t = 6$

$$\text{Gradient} = \frac{32 - 8}{10 - 2} = \frac{24}{8} = 3$$

So the acceleration is approximately 3 m/s^2

b) The tangent is drawn by eye to a curved graph, so its gradient (and therefore the acceleration) is only an estimate

8

A car accelerates uniformly from rest to a velocity of V m/s in 10 seconds. It then travels at a constant velocity of V m/s for 20 seconds, and then decelerates uniformly to rest in 5 seconds. The total distance travelled is 440 m.

a. Work out the value of V .

b. Work out the acceleration of the car in the first 10 seconds.

(4 marks)

$$\begin{array}{rcl}
 \text{stage 1: } \frac{1}{2} \times 10 \times V & = & 5V \\
 \text{stage 2: } 20 \times V & = & 20V \\
 \text{stage 3: } \frac{1}{2} \times 5 \times V & = & 2.5V \\
 & & 27.5V = 440 \\
 & & V = 16
 \end{array}$$

$\div 27.5$

$\div 27.5$

b)

$$\frac{16}{10} = 1.6$$

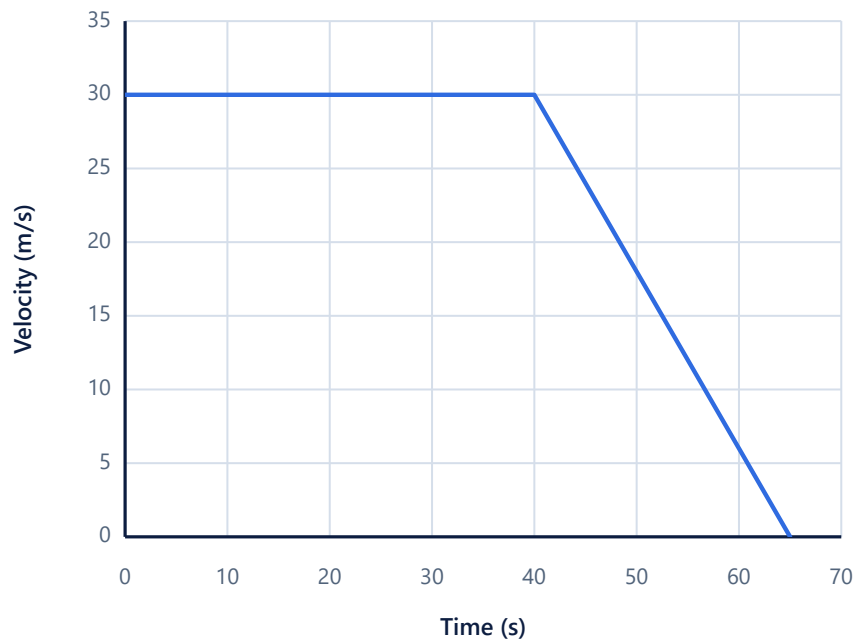
$$\boxed{1.6 \text{ m/s}^2}$$

9

A train travels at a constant velocity of 30 m/s for 40 seconds.

It then decelerates uniformly, coming to rest in a further 25 seconds.

- Work out the deceleration of the train.
- Work out the total distance the train travels in the 65 seconds.



(4 marks)

$$a) \text{ Deceleration} = \frac{30}{25} = 1.2 \text{ m/s}^2$$

$$b) \text{ Distance} = \text{area under the velocity-time graph}$$

Constant phase (rectangle)

$$40 \times 30 = 1200 \text{ m}$$

Decelerating phase (triangle)

$$\frac{1}{2} \times 25 \times 30 = 375 \text{ m}$$

$$\text{Total} = 1200 + 375 = 1575 \text{ m}$$

- 10 The velocity, v m/s, of a runner is measured every 2 seconds.

Time, t (seconds)	0	2	4	6	8
Velocity, v (m/s)	0	6	10	12	13

- Use the values at $t = 2$ and $t = 6$ to estimate the acceleration of the runner at $t = 4$.
- Use the trapezium rule with 4 strips to estimate the distance run in the first 8 seconds.
- Is your answer to part (b) an overestimate or an underestimate? Give a reason.

(5 marks)

a) Gradient between $t = 2$ and $t = 6$

$$\frac{12 - 6}{6 - 2} = \frac{6}{4} = 1.5$$

$$1.5 \text{ m/s}^2$$

b) Trapezium rule, strip width 2

$$= \frac{1}{2} \times 2 \times [(0 + 13) + 2(6 + 10 + 12)]$$

$$= 13 + 56$$

$$69 \text{ m}$$

c) Curve is above the straight tops of the trapezia

underestimate

- 11 Car A travels at a constant velocity of 15 m/s. Car B starts from the same place at the same time. Car B accelerates uniformly from rest to 20 m/s in 10 seconds and then continues at a constant 20 m/s.

- Work out the distance travelled by each car after 10 seconds.
- Work out the time at which car B catches up with car A.

(5 marks)

a) After 10 s

$$A: 15 \times 10 = 150$$

$$B: \frac{1}{2} \times 10 \times 20 = 100$$

b) For $t > 10$

$$\begin{array}{rcl}
 & & = A: 15t \\
 B: 100 + 20(t - 10) & = & 20t - 100 \\
 20t - 100 & = & 15t \\
 -15t & & \\
 5t - 100 & = & 0 \\
 +100 & & \\
 5t & = & 100 \\
 \div 5 & & \\
 t & = & 20
 \end{array}$$

$$t = 20 \text{ seconds}$$

12

A motorbike accelerates from rest. Its speed-time graph is a smooth curve that gets less steep throughout the first 12 seconds. The table gives the speed v m/s at time t seconds.

t : 0, 4, 8, 12 v : 0, 7, 11, 13

a. Use 3 strips of equal width to estimate the distance travelled in the first 12 seconds.

b. Amir says, "The acceleration at $t = 8$ is $11 \div 8 = 1.375 \text{ m/s}^2$." Explain why Amir is wrong.

(4 marks)

a) Trapezium strips of width 4: $\frac{1}{2} \times 4 \times (0 + 7) + \frac{1}{2} \times 4 \times (7 + 11) + \frac{1}{2} \times 4 \times (11 + 13)$

$$= 14 + 36 + 48 = 98 \text{ m}$$

b) 11 divided by 8 is the gradient of the straight line from the origin to (8, 11): the average acceleration over the first 8 seconds

The acceleration at the instant $t = 8$ is the gradient of the tangent to the curve at $t = 8$. Because the curve is getting less steep, this tangent gradient is smaller than 1.375 m/s^2 , so Amir's method is wrong

13

A lorry starts from rest and accelerates uniformly for 10 seconds until it reaches a velocity of 20 m/s. It then travels at a constant velocity of 20 m/s for T seconds. The lorry travels a total distance of 700 m. Work out the total time of the journey.

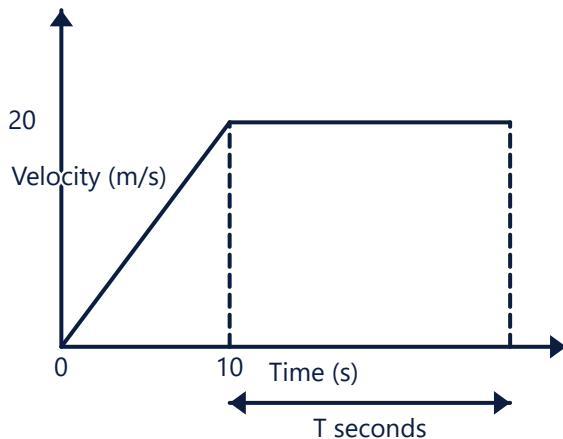


Diagram NOT accurately drawn

(5 marks)

Distance in the first 10 seconds (triangle)

$$\frac{1}{2} \times 10 \times 20 = 100 \text{ m}$$

Distance remaining at constant velocity

$$700 - 100 = 600 \text{ m}$$

Time at 20 m/s

$$\frac{600}{20} = 30 \text{ seconds, so } T = 30$$

$$\text{Total time} = 10 + 30 = 40 \text{ seconds}$$

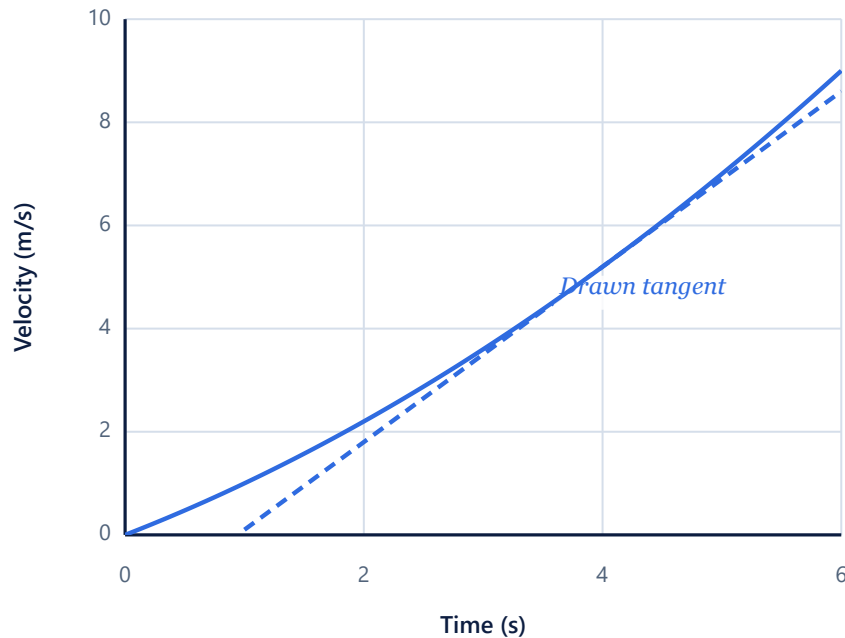
40 seconds

14

A ball rolls down a slope. Its velocity-time graph is a smooth curve. The table gives the velocity v m/s at time t seconds.

t : 0, 2, 4, 6 v : 0, 2.2, 5.2, 9.0

- A tangent drawn to the curve at $t = 4$ passes through the points (1, 0.1) and (6, 8.6). Use the tangent to estimate the acceleration of the ball at $t = 4$.
- Use the trapezium rule with 3 strips of equal width to estimate the distance the ball travels in the 6 seconds.



(5 marks)

a) Acceleration at $t = 4$ is the gradient of the tangent: $(8.6 - 0.1) \div (6 - 1) = 8.5 \div 5 = 1.7 \text{ m/s}^2$

b) Trapezium strips of width 2:

$$\frac{1}{2} \times 2 \times (0 + 2.2) = 2.2$$

$$\frac{1}{2} \times 2 \times (2.2 + 5.2) = 7.4$$

$$\frac{1}{2} \times 2 \times (5.2 + 9.0) = 14.2$$

$$\text{Estimate} = 2.2 + 7.4 + 14.2 = 23.8 \text{ m}$$

15

A car makes a journey in three stages.

Stage 1: it accelerates uniformly from rest to a velocity of V m/s in 12 seconds.

Stage 2: it travels at the constant velocity V m/s for 48 seconds.

Stage 3: it decelerates uniformly from V m/s to rest in 20 seconds.

The total distance of the journey is 960 m.

- Work out the value of V .
- Work out the acceleration of the car during stage 1.

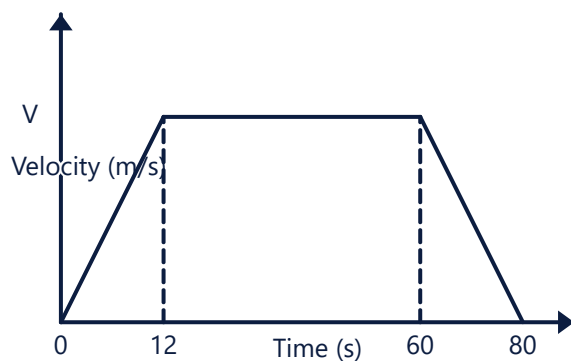


Diagram NOT accurately drawn

(6 marks)

a) Total distance = area under the velocity-time graph (a trapezium made of a triangle, a rectangle and a triangle)

Stage 1

$$\frac{1}{2} \times 12 \times V = 6V. \text{ Stage 2: } 48V. \text{ Stage 3: } \frac{1}{2} \times 20 \times V =$$

$$10V$$

$$6V + 48V + 10V = 64V = 960, \text{ so } V = 15$$

Check

$$6(15) + 48(15) + 10(15) = 90 + 720 + 150 = 960$$

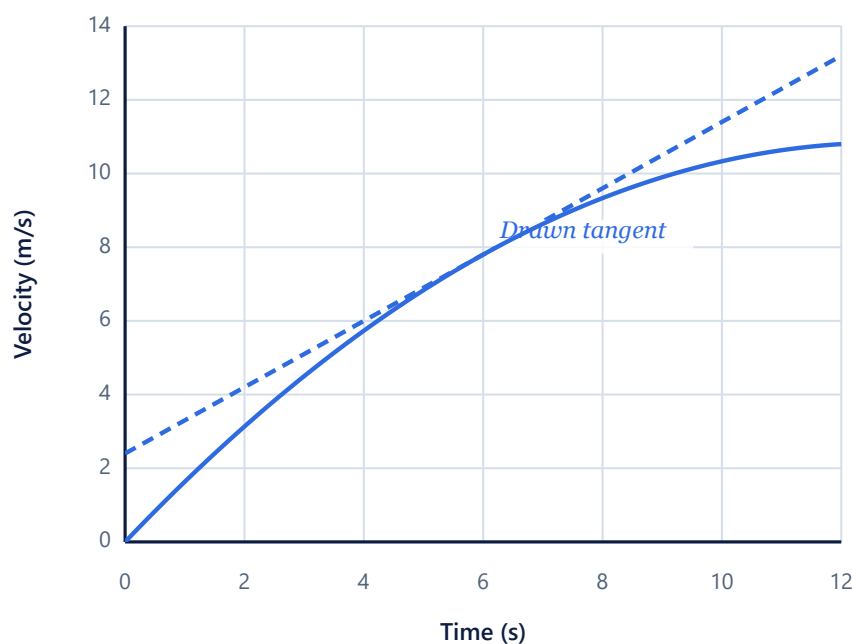
$$\text{b) Acceleration in stage 1} = \frac{15}{12} = 1.25 \text{ m/s}^2$$

16

A drone accelerates from rest. Its velocity-time graph is a smooth curve that gets less steep throughout the first 12 seconds. The table gives the velocity v m/s at time t seconds.

t : 0, 3, 6, 9, 12 v : 0, 4.5, 7.8, 9.9, 10.8

- Use the trapezium rule with 4 strips of equal width to estimate the distance travelled by the drone in the 12 seconds.
- A tangent drawn to the curve at $t = 6$ passes through the points (0, 2.4) and (12, 13.2). Use the tangent to estimate the acceleration of the drone at $t = 6$.
- Is your answer to part (a) an underestimate or an overestimate of the actual distance? Give a reason.



(7 marks)

a) Trapezium strips of width 3:

$$\frac{1}{2} \times 3 \times (0 + 4.5) = 6.75$$

$$\frac{1}{2} \times 3 \times (4.5 + 7.8) = 18.45$$

$$\frac{1}{2} \times 3 \times (7.8 + 9.9) = 26.55$$

$$\frac{1}{2} \times 3 \times (9.9 + 10.8) = 31.05$$

$$\text{Estimate} = 6.75 + 18.45 + 26.55 + 31.05 = 82.8 \text{ m}$$

b) Gradient of the tangent = $(13.2 - 2.4) \div (12 - 0) = 10.8 \div 12 = 0.9$, so the acceleration at $t = 6$ is about 0.9 m/s^2

c) The question states that the curve gets less steep throughout the interval. Each straight top edge of a trapezium lies below the curve, so the trapezia miss some area: the estimate is an underestimate. The table values alone would not prove this shape between the recorded times