

Volume of cones, spheres and pyramids

Worked solutions

GCSE Maths topic practice



This topic online

- 1 A cone has base radius 6 cm and vertical height 10 cm.

Work out the volume of the cone. Give your answer correct to 3 significant figures.

(2 marks)

$$\begin{aligned} V &= \frac{1}{3} \times \pi \times r^2 \times h \\ &= \frac{1}{3} \times \pi \times 6^2 \times 10 \\ &= 120\pi \end{aligned}$$

$$\boxed{377 \text{ cm}^3 \text{ (3 s.f.)}}$$

- 2 A sphere has radius 3 cm.

- Work out the volume of the sphere. Give your answer in terms of π .
- Write your answer to part (a) as a number, correct to 3 significant figures.

(3 marks)

a)

$$\begin{aligned} V &= \frac{4}{3} \times \pi \times r^3 \\ &= \frac{4}{3} \times \pi \times 3^3 \\ &= \frac{4}{3} \times \pi \times 27 \end{aligned}$$

$$\boxed{36\pi \text{ cm}^3}$$

b)

$$36\pi = 113.09\dots$$

$$\boxed{113 \text{ cm}^3}$$

- 3 A pyramid has a square base of side 8 cm. Its perpendicular height is 9 cm.
Work out the volume of the pyramid.

(2 marks)

$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$\text{base area} = 8 \times 8 = 64$$

$$V = \frac{1}{3} \times 64 \times 9$$

$$\boxed{192 \text{ cm}^3}$$

- 4 A solid hemisphere has radius 6 cm.
Work out the volume of the hemisphere.
Give your answer correct to 3 significant figures.

(2 marks)

$$\text{Volume of a sphere} = \frac{4}{3} \times \pi \times r^3, \text{ so volume of a hemisphere} = \frac{2}{3} \times \pi \times r^3$$

$$\text{Volume} = \frac{2}{3} \times \pi \times 6^3 = \frac{2}{3} \times \pi \times 216 = 144\pi$$

$$144\pi = 452.38... = 452 \text{ cm}^3 \text{ (3 sf)}$$

$$\boxed{452 \text{ cm}^3}$$

5

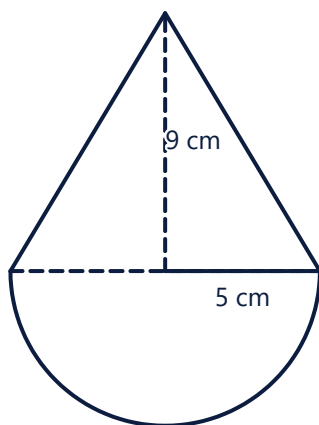
A solid is made from a cone and a hemisphere.

The cone has base radius 5 cm and vertical height 9 cm.

The flat face of the hemisphere, of radius 5 cm, is joined to the base of the cone.

Work out the total volume of the solid.

Give your answer correct to 3 significant figures.



Central vertical cross-section

Diagram NOT accurately drawn

(4 marks)

$$\text{Volume of the cone} = \frac{1}{3} \times \pi \times 5^2 \times 9 = \frac{1}{3} \times \pi \times 225 = 75\pi \text{ cm}^3$$

$$\text{Volume of the hemisphere} = \frac{2}{3} \times \pi \times 5^3 = \frac{250\pi}{3} \text{ cm}^3$$

$$\text{Total volume} = 75\pi + \frac{250\pi}{3} = \frac{475\pi}{3} = 497.41\dots \text{ cm}^3$$

$$\text{Total volume} = 497 \text{ cm}^3 \text{ (3 s.f.)}$$

$$\boxed{497 \text{ cm}^3}$$

6

An ice cream cone has base radius 5 cm and volume $250\pi \text{ cm}^3$.

Work out the vertical height of the cone.

(3 marks)

$$\begin{array}{rcl} \frac{1}{3} \times \pi \times 5^2 \times h & = & 250\pi \\ \div \pi & & \div \pi \\ \frac{25}{3} h & = & 250 \\ \times 3 & & \times 3 \\ 25h & = & 750 \\ \div 25 & & \div 25 \\ h & = & 30 \end{array}$$

$$\boxed{30 \text{ cm}}$$

7

A solid metal ball has radius 5 cm. The metal has density 7.8 g/cm^3 .

Work out the mass of the ball in kilograms. Give your answer correct to 3 significant figures.

(4 marks)

$$V = \frac{4}{3} \times \pi \times 5^3 = 523.59... \text{ cm}^3$$

$$\text{mass} = \text{density} \times \text{volume}$$

$$= 7.8 \times 523.59...$$

$$= 4084 \text{ g}$$

$$\boxed{4.08 \text{ kg (3 s.f.)}}$$

8

A solid is a quarter of a sphere. It is made by cutting a sphere in half, then cutting one hemisphere in half again.

The volume of the solid is $243\pi \text{ cm}^3$.

Work out the total surface area of the solid.

Give your answer in terms of π .

Both cuts pass through the centre of the original sphere, and the second cutting plane is perpendicular to the first.

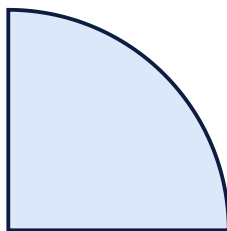


Diagram NOT accurately drawn

Section perpendicular to both cuts

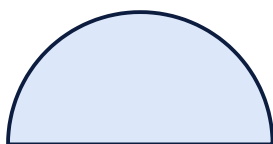


Diagram NOT accurately drawn

Flat faces, shown separately

(5 marks)

$$\text{Volume of a quarter sphere} = \frac{1}{4} \times \frac{4}{3} \times \pi \times r^3 = \frac{1}{3} \times \pi \times r^3$$

$$\frac{1}{3} \times \pi \times r^3 = 243\pi, \text{ so } r^3 = 729 \text{ and } r = 9 \text{ cm}$$

$$\text{Curved part} = \frac{1}{4} \text{ of the sphere's surface} = \frac{1}{4} \times 4\pi r^2 = \pi r^2 = 81\pi \text{ cm}^2$$

$$\text{The two flat faces are semicircles of radius 9, each of area } \frac{1}{2}\pi r^2, \text{ together } \pi r^2 = 81\pi \text{ cm}^2$$

$$\text{Total surface area} = 81\pi + 81\pi = 162\pi \text{ cm}^2$$

$$\boxed{162\pi \text{ cm}^2}$$

9

A solid cone has base radius 5 cm.

The curved surface area of the cone is $65\pi \text{ cm}^2$.

Work out the volume of the cone.

Give your answer correct to 3 significant figures.

(5 marks)

$$\text{Curved surface area} = \pi r l, \text{ so } \pi \times 5 \times l = 65\pi \text{ and } l = 13 \text{ cm}$$

$$\text{By Pythagoras, height} = \sqrt{(13^2 - 5^2)} = \sqrt{144} = 12 \text{ cm}$$

$$\text{Volume} = \frac{1}{3} \times \pi \times r^2 \times h = \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi$$

$$100\pi = 314.15... = 314 \text{ cm}^3 \text{ (3 s.f.)}$$

$$\boxed{314 \text{ cm}^3}$$

10

A solid hemisphere has volume $18\pi \text{ cm}^3$.

Work out the total surface area of the hemisphere. Give your answer in terms of π .

(4 marks)

$$\frac{2}{3} \times \pi \times r^3 = 18\pi$$

$$\times \frac{3}{2}, \div \pi$$

$$r^3 = 27$$

$$r = 3$$

$$\times \frac{3}{2}, \div \pi$$

$$\text{curved surface} = 2\pi r^2 = 18\pi$$

$$\text{flat face} = \pi r^2 = 9\pi$$

$$= 18\pi + 9\pi$$

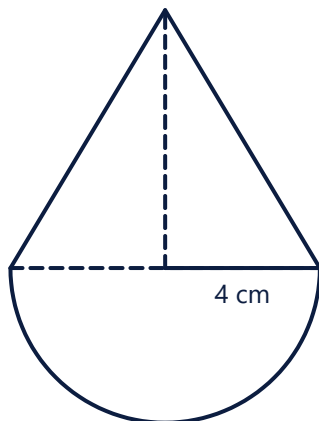
$$\boxed{27\pi \text{ cm}^2}$$

11

A solid is made from a cone and a hemisphere, both of radius 4 cm. The flat face of the hemisphere is joined to the base of the cone.

The total volume of the solid is $96\pi \text{ cm}^3$.

Work out the total height of the solid.



Central vertical cross-section

Diagram NOT accurately drawn

(5 marks)

Let h be the vertical height of the cone

$$\text{Volume of the cone} = \frac{1}{3} \times \pi \times 16 \times h = \frac{16\pi h}{3}$$

$$\text{Volume of the hemisphere} = \frac{2}{3} \times \pi \times 64 = \frac{128\pi}{3}$$

$$\frac{16\pi h}{3} + \frac{128\pi}{3} = 96\pi, \text{ so } 16h + 128 = 288$$

$$16h = 160, \text{ so } h = 10 \text{ cm}$$

$$\text{Total height} = \text{height of cone} + \text{radius of hemisphere} = 10 + 4 = 14 \text{ cm}$$

14 cm

12

A solid is made from a cylinder and a cone. Both have radius 4 cm. The cylinder has height 10 cm. The cone sits on top of the cylinder and has vertical height h cm.

The total volume of the solid is $224\pi \text{ cm}^3$.

Work out the value of h .

(4 marks)

$$\begin{array}{rcl}
 \text{cylinder} & = & \pi \times 4^2 \times 10 = 160\pi \\
 \text{cone} & = & \frac{1}{3} \times \pi \times 4^2 \times h = \\
 & & \frac{16\pi}{3} h \\
 160\pi + \frac{16\pi}{3} h & = & 224\pi \\
 -160\pi, \div \pi & & -160\pi, \div \pi \\
 \frac{16}{3} h & = & 64 \\
 \times 3 & & \times 3 \\
 16h & = & 192 \\
 \div 16 & & \div 16 \\
 h & = & 12
 \end{array}$$

$$h = 12$$

13

A cube has sides of length 6 cm.

A pyramid has a square base of side 9 cm and height h cm.

Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

The volume of the pyramid is equal to the volume of the cube.

Work out the value of h .

(5 marks)

$$\text{Volume of the cube} = 6 \times 6 \times 6 = 216 \text{ cm}^3$$

$$\text{Volume of the pyramid} = \frac{1}{3} \times 9 \times 9 \times h = 27h \text{ cm}^3$$

$$27h = 216, \text{ so } h = 216 \div 27 = 8$$

$$8$$

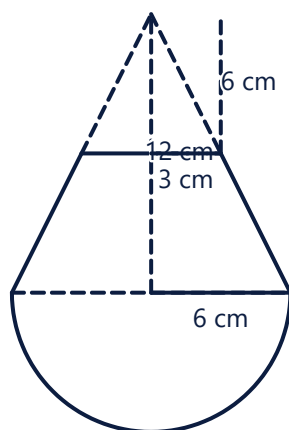
14

A frustum is made by removing a cone of base radius 3 cm and vertical height 6 cm from a similar cone of base radius 6 cm and vertical height 12 cm.

A solid hemisphere of radius 6 cm is attached to the larger face of the frustum to make a solid.

Calculate the total volume of the solid.

Give your answer correct to 3 significant figures.



Central vertical cross-section

Diagram NOT accurately drawn

(5 marks)

$$\text{Volume of the large cone} = \frac{1}{3} \times \pi \times 36 \times 12 = 144\pi \text{ cm}^3$$

$$\text{Volume of the removed cone} = \frac{1}{3} \times \pi \times 9 \times 6 = 18\pi \text{ cm}^3$$

$$\text{Volume of the frustum} = 144\pi - 18\pi = 126\pi \text{ cm}^3$$

$$\text{Volume of the hemisphere} = \frac{2}{3} \times \pi \times 216 = 144\pi \text{ cm}^3$$

$$\text{Total volume} = 126\pi + 144\pi = 270\pi = 848.23\dots = 848 \text{ cm}^3$$

(3 s.f.)

$$\boxed{848 \text{ cm}^3}$$

15

A cone has base radius 9 cm and vertical height 15 cm.

A smaller cone of vertical height 5 cm is cut off the top, using a cut parallel to the base. The solid left is a frustum.

Work out the volume of the frustum. Give your answer correct to 3 significant figures.

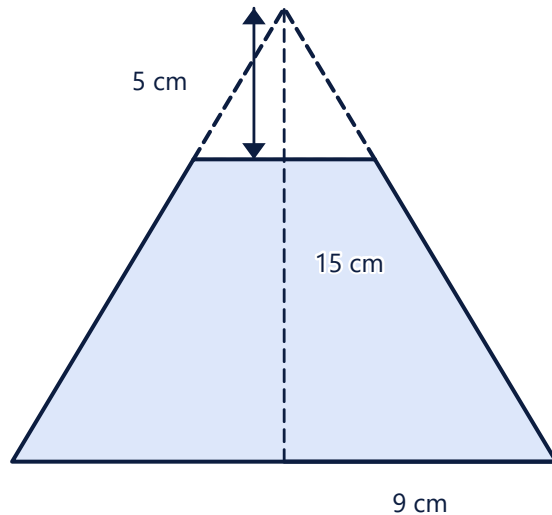


Diagram NOT accurately drawn

(5 marks)

$$\text{scale factor} = \frac{5}{15} = \frac{1}{3}$$

$$\text{small radius} = 9 \times \frac{1}{3} = 3$$

$$\text{big cone} = \frac{1}{3} \times \pi \times 9^2 \times 15 = 405\pi$$

$$\text{small cone} = \frac{1}{3} \times \pi \times 3^2 \times 5 = 15\pi$$

$$\text{frustum} = 405\pi - 15\pi = 390\pi$$

$$\boxed{1230 \text{ cm}^3 \text{ (3 s.f.)}}$$

- 16 A solid cone has base radius 6 cm and volume $96\pi \text{ cm}^3$.

The curved surface of the cone is made from a sector of a circle.

Work out the angle of the sector.

(5 marks)

$$\text{Volume} = \frac{1}{3} \times \pi \times 36 \times h = 96\pi, \text{ so } 12h = 96 \text{ and } h = 8 \text{ cm}$$

$$\text{Slant height } l = \sqrt{(8^2 + 6^2)} = \sqrt{100} = 10 \text{ cm}$$

The sector has radius $l = 10 \text{ cm}$, and its arc length equals the circumference of the base, $2\pi \times 6 = 12\pi \text{ cm}$

$$\text{Angle} \div 360 = \text{arc} \div \text{full circumference} = 12\pi \div 20\pi = \frac{3}{5}$$

$$\text{Angle} = \frac{3}{5} \times 360 = 216^\circ$$

216°

- 17 A solid sphere has radius r . A solid cone has base radius r and vertical height h .

The volume of the cone is equal to the volume of the sphere.

a. Show that $h = 4r$.

b. Find the ratio of the curved surface area of the cone to the surface area of the sphere. Give your answer in the form $\sqrt{k} : m$.

(6 marks)

$$\text{a) Volume of cone} = \frac{1}{3} \times \pi \times r^2 \times h \text{ and volume of sphere} = \frac{4}{3} \times \pi \times r^3$$

Setting them equal

$$\frac{1}{3} \pi r^2 h = \frac{4}{3} \pi r^3, \text{ so } h = 4r \text{ as required}$$

$$\text{b) Slant height } l = \sqrt{(r^2 + h^2)} = \sqrt{(r^2 + 16r^2)} = \sqrt{(17r^2)} = r\sqrt{17}$$

$$\text{Curved surface area of the cone} = \pi r l = \sqrt{17} \times \pi r^2$$

$$\text{Surface area of the sphere} = 4\pi r^2$$

$$\text{Ratio} = \sqrt{17}\pi r^2 : 4\pi r^2 = \sqrt{17} : 4$$

18

A flower pot is in the shape of a frustum. The diagram shows its cross-section. The top is a circle of radius 9 cm, the bottom is a circle of radius 3 cm and the height is 8 cm.

The pot is a frustum, so it is a large cone with a small cone cut off the bottom.

- Show that the cone that was cut off has height 4 cm.
- Work out the volume of the pot. Give your answer correct to 3 significant figures.

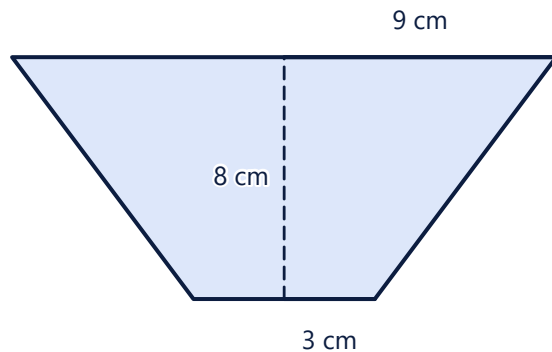


Diagram NOT accurately drawn

(4 marks)

a)

Similar cones: radii $3 : 9 = 1 : 3$

$$\frac{h}{h+8} = \frac{1}{3}$$

$$3h = h + 8$$

$$h = 4 \text{ cm}$$

b)

$$\text{big cone: } \frac{1}{3} \times \pi \times 9^2 \times 12 = 324\pi$$

$$\text{small cone: } \frac{1}{3} \times \pi \times 3^2 \times 4 = 12\pi$$

$$324\pi - 12\pi = 312\pi$$

$$980 \text{ cm}^3 \text{ (3 s.f.)}$$

19

A toy is made from a solid cylinder of wood with a solid hemisphere of plastic stuck on top. The diagram shows its cross-section. The cylinder and the hemisphere both have radius 4 cm. The cylinder has height 10 cm.

The wood has density 0.6 g/cm^3 . The plastic has density 1.2 g/cm^3 .

Work out the average density of the whole toy. Give your answer in g/cm^3 correct to 3 significant figures.

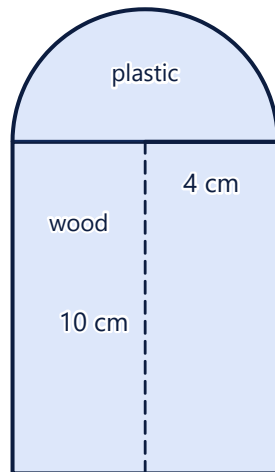


Diagram NOT accurately drawn

(5 marks)

$$\text{cylinder: } \pi \times 4^2 \times 10 = 160\pi$$

$$\text{hemisphere: } \frac{2}{3} \times \pi \times 4^3 = \frac{128}{3}\pi$$

$$\text{wood mass: } 0.6 \times 160\pi = 96\pi$$

$$\text{plastic mass: } 1.2 \times \frac{128}{3}\pi = 51.2\pi$$

$$\text{total mass} = 147.2\pi$$

$$\text{total volume} = 202.67\pi$$

density = mass $\div$ volume

$$= \frac{147.2\pi}{202.67\pi}$$

$$\boxed{0.726 \text{ g/cm}^3 \text{ (3 s.f.)}}$$