



1 Simplify fully $\frac{2x^2 + 6x}{4x}$

(2 marks)

Factorise the top

$$\begin{aligned} 2x^2 + 6x &= 2x(x + 3) \\ &= \frac{2x(x + 3)}{4x} \end{aligned}$$

Cancel the common factor $2x$

$$\frac{x + 3}{2}$$

2 Simplify fully

$$\frac{x^2 + 5x}{x^2 + 8x + 15}$$

(3 marks)

The original expression is defined only when $x \neq -5$ and $x \neq -3$. A denominator cannot be zero. These restrictions still apply after simplifying

Factorise the numerator

$$x^2 + 5x = x(x + 5)$$

Factorise the denominator

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

Cancel the common factor $(x + 5)$ to get $\frac{x}{x + 3}$

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\frac{x}{x + 3}$$

3

Simplify fully

$$\frac{x^2 - 9}{2x^2 + 5x - 3}$$

(3 marks)

The original expression is defined only when $x \neq -3$ and $x \neq 1/2$. A denominator cannot be zero. These restrictions still apply after simplifying

Factorise the numerator using the difference of two squares

$$x^2 - 9 = (x - 3)(x + 3)$$

Factorise the denominator

$$2x^2 + 5x - 3 = (2x - 1)(x + 3)$$

Cancel the common factor $(x + 3)$ to get $\frac{x - 3}{2x - 1}$

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\boxed{\frac{x - 3}{2x - 1}}$$

4

Simplify fully $\frac{4x^2 - 9}{2x^2 + x - 6}$

(3 marks)

Factorise the top (difference of two squares)

$$4x^2 - 9 = (2x - 3)(2x + 3)$$

Factorise the bottom

$$\begin{aligned} 2x^2 + x - 6 &= (2x - 3)(x + 2) \\ &= \frac{(2x - 3)(2x + 3)}{(2x - 3)(x + 2)} \end{aligned}$$

Cancel $(2x - 3)$

$$\boxed{\frac{2x + 3}{x + 2}}$$

5

Write

$$\frac{3}{x+2} + \frac{5}{x-1}$$

as a single fraction in its simplest form.

(3 marks)

The original expression is defined only when $x \neq -2$ and $x \neq 1$. A denominator cannot be zero. These restrictions still apply after simplifying

Make the denominators match by multiplying each fraction's numerator and denominator by the same missing factor. This multiplies each fraction by 1, so its value stays the same

Use the common denominator $(x+2)(x-1)$

$$\frac{3}{x+2} + \frac{5}{x-1} = \frac{3(x-1) + 5(x+2)}{(x+2)(x-1)}$$

Expand the numerator

$$3x - 3 + 5x + 10 = 8x + 7$$

So the single fraction is $\frac{8x+7}{(x+2)(x-1)}$

$$\boxed{\frac{8x+7}{(x+2)(x-1)}}$$

6

Show that $\frac{x+1}{x-1} - \frac{x-1}{x+1}$ can be written as $\frac{4x}{x^2-1}$.

(3 marks)

Common denominator $(x-1)(x+1)$

$$= \frac{(x+1)^2 - (x-1)^2}{(x-1)(x+1)}$$

Expand the top

$$= \frac{x^2 + 2x + 1 - x^2 + 2x - 1}{(x-1)(x+1)}$$

$$= \frac{4x}{(x-1)(x+1)}$$

Denominator is a difference of two squares

$$\boxed{\frac{4x}{x^2-1}}$$

7

Show that

$$\frac{4}{x+3} - \frac{2}{x+5}$$

can be written as $\frac{2(x+7)}{(x+3)(x+5)}$

(3 marks)

The original expression is defined only when $x \neq -3$ and $x \neq -5$. A denominator cannot be zero. These restrictions still apply after simplifying

Make the denominators match by multiplying each fraction's numerator and denominator by the same missing factor. This multiplies each fraction by 1, so its value stays the same

Use the common denominator $(x+3)(x+5)$

$$\frac{4}{x+3} - \frac{2}{x+5} = \frac{4(x+5) - 2(x+3)}{(x+3)(x+5)}$$

Expand the numerator

$$4x + 20 - 2x - 6 = 2x + 14$$

Factorise

$$2x + 14 = 2(x+7), \text{ giving } \frac{2(x+7)}{(x+3)(x+5)} \text{ as required}$$

$$\frac{2(x+7)}{(x+3)(x+5)}$$

8

Simplify fully

$$\left[\frac{x^2 + 7x + 10}{x^2 - 4} \right] \div \left[\frac{x^2 + 10x + 25}{3x - 6} \right]$$

(4 marks)

The original expression is defined only when $x \neq 2, -2$ or -5 . A denominator cannot be zero, and the fraction we divide by cannot be zero. These restrictions still apply after simplifying

Factorise each expression

$$x^2 + 7x + 10 = (x + 2)(x + 5), \quad x^2 - 4 = (x - 2)(x + 2), \quad x^2 + 10x + 25 = (x + 5)^2, \quad 3x - 6 = 3(x - 2)$$

Dividing by a fraction is multiplying by its reciprocal: $\left[\frac{(x + 2)(x + 5)}{(x - 2)(x + 2)} \right] \times \left[\frac{3(x - 2)}{(x + 5)^2} \right]$

Cancel $(x + 2)$: gives $\frac{x + 5}{x - 2} \times \frac{3(x - 2)}{(x + 5)^2}$

Cancel $(x - 2)$ and one factor of $(x + 5)$ to leave $\frac{3}{x + 5}$

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\boxed{\frac{3}{x+5}}$$

9

Show that $\frac{2x^2 - 8}{x - 2} \div \frac{x + 2}{x}$ simplifies to $2x$.

(4 marks)

Factorise the top of the first fraction

$$\begin{aligned} 2x^2 - 8 &= 2(x - 2)(x + 2) \\ &= \frac{2(x - 2)(x + 2)}{x - 2} \div \frac{x + 2}{x} \end{aligned}$$

Cancel $(x - 2)$ on the first fraction

$$= 2(x + 2) \div \frac{x + 2}{x}$$

Dividing by a fraction: flip it and multiply

$$= 2(x + 2) \times \frac{x}{x + 2}$$

Cancel $(x + 2)$

$$\boxed{2x}$$

10 Simplify fully

$$\left[\frac{2x^2 + 7x + 3}{x^2 - 9} \right] \times \left[\frac{x^2 - 3x}{4x^2 - 1} \right]$$

(4 marks)

The original expression is defined only when $x \neq 3, -3, 1/2$ or $-1/2$. A denominator cannot be zero. These restrictions still apply after simplifying

Factorise each expression

$$2x^2 + 7x + 3 = (2x + 1)(x + 3), \quad x^2 - 9 = (x - 3)(x + 3), \quad x^2 - 3x = x(x - 3), \quad 4x^2 - 1 = (2x - 1)(2x + 1)$$

$$\text{The product is } \frac{(2x + 1)(x + 3) \times x(x - 3)}{(x - 3)(x + 3)(2x - 1)(2x + 1)}$$

Cancel $(2x + 1)$, $(x + 3)$ and $(x - 3)$

$$\text{This leaves } \frac{x}{2x - 1}$$

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\boxed{\frac{x}{2x - 1}}$$

11 Solve $\frac{3}{x + 1} + \frac{2}{x - 2} = 1$

Give your answers in the form $a \pm \sqrt{b}$, where a and b are integers.

(5 marks)

$$\frac{3}{x + 1} + \frac{2}{x - 2} = 1$$

Multiply every term by $(x + 1)(x - 2)$

$$3(x - 2) + 2(x + 1) = (x + 1)(x - 2)$$

Expand both sides

$$5x - 4 = x^2 - x - 2$$

Rearrange to equal 0

$$x^2 - 6x + 2 = 0$$

Complete the square

$$(x - 3)^2 - 7 = 0$$

$$(x - 3)^2 = 7$$

$$x - 3 = \pm\sqrt{7}$$

$$\boxed{x = 3 + \sqrt{7} \text{ or } x = 3 - \sqrt{7}}$$

12 Simplify fully

$$\left(1 + \frac{2}{x}\right) \div \left(1 - \frac{4}{x^2}\right)$$

(4 marks)

The original expression is defined only when $x \neq 0, 2$ or -2 . A denominator cannot be zero, and the fraction we divide by cannot be zero. These restrictions still apply after simplifying

Write each bracket as a single fraction

$$1 + \frac{2}{x} = \frac{x+2}{x} \text{ and } 1 - \frac{4}{x^2} = \frac{x^2-4}{x^2}$$

$$\text{Dividing gives } \frac{x+2}{x} \times \frac{x^2}{x^2-4}$$

$$\text{Factorise } x^2 - 4 = (x-2)(x+2)$$

$$\text{Cancel } (x+2) \text{ and one factor of } x \text{ to leave } \frac{x}{x-2}$$

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\boxed{\frac{x}{x-2}}$$

13 Write $\frac{1}{x} + \frac{2}{x+1} + \frac{3}{x+2}$ as a single fraction.

Expand and simplify the numerator and the denominator.

(5 marks)

Common denominator $x(x+1)(x+2)$

$$= \frac{(x+1)(x+2) + 2x(x+2) + 3x(x+1)}{x(x+1)(x+2)}$$

Expand each part of the top

$$(x+1)(x+2) = x^2 + 3x + 2$$

$$2x(x+2) = 2x^2 + 4x$$

$$3x(x+1) = 3x^2 + 3x$$

Add them

$$\text{top} = 6x^2 + 10x + 2$$

Expand the bottom: $x(x+1)(x+2) = x(x^2 + 3x + 2)$

$$\boxed{\frac{6x^2 + 10x + 2}{x^3 + 3x^2 + 2x}}$$

14

Show that

$$\frac{x^2 + 3x}{x^2 - x - 12} - \frac{2}{x + 1}$$

can be written as $\frac{x^2 - x + 8}{(x - 4)(x + 1)}$

(6 marks)

The original expression is defined only when $x \neq -3, 4$ or -1 . A denominator cannot be zero. These restrictions still apply after simplifying

Factorise the first fraction

$$x^2 + 3x = x(x + 3) \text{ and } x^2 - x - 12 = (x - 4)(x + 3)$$

Cancel $(x + 3)$: the first fraction simplifies to $\frac{x}{x - 4}$

Combine over the common denominator $(x - 4)(x + 1)$:

$$\frac{x(x + 1) - 2(x - 4)}{(x - 4)(x + 1)}$$

Expand the numerator

$$x^2 + x - 2x + 8 = x^2 - x + 8$$

This gives $\frac{x^2 - x + 8}{(x - 4)(x + 1)}$ as required

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\boxed{\frac{x^2 - x + 8}{(x - 4)(x + 1)}}$$