Arithmetic series: nth term and sum

Worked solutions

GCSE Maths topic practice

This topic online

1 Here is an arithmetic series.

6 + 11 + 16 + 21 + 26 + ...

a. Write down the first term, a.

b. Write down the common difference, d.

(2 marks)

a)

$a = 6$

b)

$d = 11 - 6$

$d = 5$

2 An arithmetic sequence has first term 3 and common difference 7.

Find the 20th term of the sequence.

(2 marks)

nth term $= a + (n - 1)d$

$u20 = 3 + 19 \times 7$

$= 3 + 133$

136

3 The first four terms of an arithmetic sequence are

40, 37, 34, 31, ...

Find the 25th term of the sequence.

(2 marks)

$a = 40$, $d = 37 - 40 = -3$

$u25 = 40 + 24 \times (-3)$

$= 40 - 72$

-32

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4 The first four terms of an arithmetic series are

3 + 8 + 13 + 18 + ...

Find the sum of the first 40 terms of the series.

You may use $S_n = \frac{n}{2}[2a + (n - 1)d]$

(3 marks)

$a = 3, d = 5, n = 40$

$S_{40} = \frac{40}{2}[2 \times 3 + 39 \times 5]$

$= 20 \times [6 + 195]$

$= 20 \times 201$

$\boxed{4020}$

5 In an arithmetic sequence, the 3rd term is 11 and the 8th term is 31.

a. Find the first term, a.

b. Find the common difference, d.

c. Find the sum of the first 20 terms of the sequence.

(5 marks)

a) and b)

$a + 2d = 11$

$a + 7d = 31$

Subtract the first from the second

$5d = 20$

$\div 5$ $\quad d = 4$ $\quad \div 5$

$a + 2 \times 4 = 11$

$a + 8 = 11$

-8 $\quad a = 3$ $\quad -8$

$\boxed{a = 3, d = 4}$

c)

$S_{20} = \frac{20}{2}[2 \times 3 + 19 \times 4]$

$= 10 \times [6 + 76]$

$= 10 \times 82$

$\boxed{820}$

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6 A theatre has 25 rows of seats.

There are 18 seats in the first row, 21 seats in the second row, 24 seats in the third row, and so on.
Each row has 3 more seats than the row in front of it.

a. Work out the number of seats in the back row.

b. Work out the total number of seats in the theatre.

(4 marks)

$a = 18, d = 3, n = 25$

a)

$u_{25} = 18 + 24 \times 3$

$= 18 + 72$

90 seats

b)

$S_{25} = \frac{25}{2}[2 \times 18 + 24 \times 3]$

$= 12.5 \times [36 + 72]$

$= 12.5 \times 108$

1350 seats

7 The first three terms of an arithmetic sequence are

7, 12, 17, ...

a. Find the value of n for which the nth term is the first term of the sequence that is greater than 300.

b. Write down the value of this term.

(3 marks)

$a = 7, d = 5$

a)

$7 + 5(n - 1) > 300$

$5(n - 1) > 293$

$n - 1 > 58.6$

$n > 59.6$

n must be a whole number

$n = 60$

b)

$u_{60} = 7 + 59 \times 5$

302

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8 Priya is saving for a bicycle.

She saves £5 in the first week, £7 in the second week, £9 in the third week, and so on. Each week she saves £2 more than the week before.

a. Work out the amount, in pounds, that Priya saves in week 12.

b. Work out the total amount, in pounds, that Priya has saved by the end of week 12.

c. Priya needs more than £1000 for the bicycle. Work out the number of weeks it takes for her total savings to first exceed £1000.

(5 marks)

$a = 5, d = 2$

a)

$u_{12} = 5 + 11 \times 2$

£27

b)

$S_{12} = \frac{12}{2}[2 \times 5 + 11 \times 2]$

$= 6 \times 32$

£192

c)

$S_n = \frac{n}{2}[10 + 2(n-1)]$

$= \frac{n}{2}(2n + 8)$

$= n(n + 4)$

$n(n + 4) > 1000$

Solve $n^2 + 4n - 1000 = 0$

$n = \frac{-4 + \sqrt{(16 + 4000)}}{2}$

$= 29.68...$

First whole number of weeks above 29.68

Check: $29 \times 33 = 957$, $30 \times 34 = 1020$

30 weeks

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9 The first three terms of an arithmetic series are

5 + 8 + 11 + ...

The sum of the first n terms of the series is 1455.

Find the value of n.

(4 marks)

$a = 5, d = 3$

$$\frac{n}{2}[2 \times 5 + (n-1) \times 3] = 1455$$

$$\frac{n}{2}(3n + 7) = 1455$$

$\times 2$ $\quad 3n^2 + 7n = 2910$ $\quad \times 2$

-2910 $\quad 3n^2 + 7n - 2910 = 0$ $\quad -2910$

$$n = \frac{-7 \pm \sqrt{(49 + 34920)}}{6}$$

$$= \frac{-7 \pm 187}{6}$$

$n = 30$ or $n = -32.3...$

n must be a positive whole number

$\boxed{n = 30}$

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10 The first three terms of an arithmetic series are

4 + 10 + 16 + ...

a. Find the 11th term of the series.

b. Find the sum of the 11th term to the 30th term inclusive.

(5 marks)

$a = 4,\ d = 6$

a)

$u_{11} = 4 + 10 \times 6$

$\boxed{64}$

b) 11th to 30th = $S_{30} - S_{10}$

$$S_{30} = \frac{30}{2}[2 \times 4 + 29 \times 6]$$

$$= 15 \times 182 = 2730$$

$$S_{10} = \frac{10}{2}[2 \times 4 + 9 \times 6]$$

$$= 5 \times 62 = 310$$

$$S_{30} - S_{10} = 2730 - 310$$

$\boxed{2420}$

Check: 20 terms, $\frac{20}{2}(64 + 178) = 2420$

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