



1

A, B, C and D are points on a circle, in that order round the circumference, so ABCD is a cyclic quadrilateral.

Angle DAB =  $108^\circ$  and angle ABC =  $75^\circ$ .

- Work out the size of angle BCD.
- Work out the size of angle CDA.

Give a reason for each answer.

(3 marks)

a)

BCD is opposite DAB

$$\begin{array}{rcl} \text{BCD} + 108^\circ & = & 180^\circ \\ -108^\circ & & \\ \hline \text{BCD} & = & 72^\circ \end{array} \quad -108^\circ$$

Reason: opposite angles in a cyclic quadrilateral add up to  $180^\circ$

b)

CDA is opposite ABC

$$\begin{array}{rcl} \text{CDA} + 75^\circ & = & 180^\circ \\ -75^\circ & & \\ \hline \text{CDA} & = & 105^\circ \end{array} \quad -75^\circ$$

Reason: opposite angles in a cyclic quadrilateral add up to  $180^\circ$

a)  $72^\circ$  b)  $105^\circ$

2

A, B, C and D are points on a circle, in that order round the circumference. The chords AC and BD cross at X.

Angle ABD =  $34^\circ$  and angle CAB =  $52^\circ$ .

- Work out the size of angle ACD.
- Work out the size of angle BDC.

Give a reason for each answer.

(3 marks)

a)

B and C are both on the same side of chord AD

$$\angle ACD = \angle ABD = 34^\circ$$

Reason: angles in the same segment are equal

b)

A and D are both on the same side of chord CB

$$\angle BDC = \angle CAB = 52^\circ$$

Reason: angles in the same segment are equal

a)  $34^\circ$  b)  $52^\circ$

3

A, B and C are points on a circle with centre O.

AOC is a diameter of the circle.

Angle CAB =  $37^\circ$ .

Work out the size of angle ACB. Give a reason for each stage of your working.

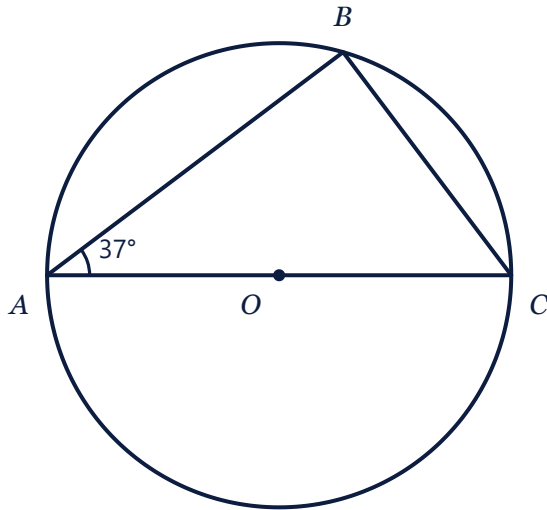


Diagram NOT accurately drawn

(3 marks)

Angle ABC =  $90^\circ$  because the angle in a semicircle is  $90^\circ$

Angles in a triangle add up to  $180^\circ$

Angle ACB =  $180^\circ - 90^\circ - 37^\circ = 53^\circ$

53

4

A, B and C are points on a circle with centre O.

B lies on the major arc AC.

Angle AOC =  $128^\circ$ .

Work out the size of angle ABC. Give a reason for your answer.

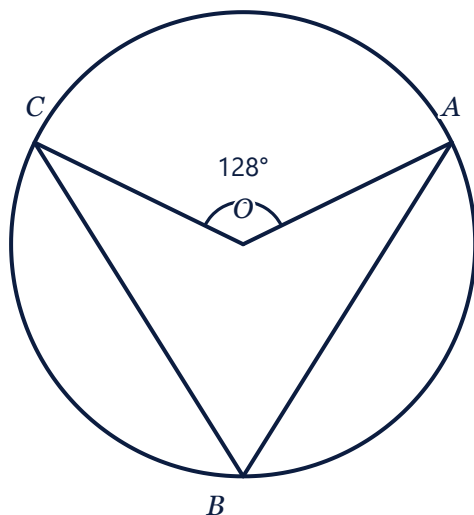


Diagram NOT accurately drawn

(3 marks)

The angle at the centre of a circle is twice the angle at the circumference when both stand on the same arc

$$\text{Angle } ABC = 128^\circ \div 2 = 64^\circ$$

64

5

A, B and C are points on a circle. The line PAQ is the tangent to the circle at A. B and C are on the same side of PQ, with B nearer to P.

Angle PAB =  $64^\circ$  and angle BAC =  $50^\circ$ .

- Work out the size of angle ABC.
- Work out the size of angle CAQ.

Give a reason for each stage of your working.

(4 marks)

a)

$$\angle ACB = \angle PAB = 64^\circ$$

Reason: alternate segment theorem

$$\angle ABC = 180^\circ - 64^\circ - 50^\circ$$

$$\angle ABC = 66^\circ$$

Reason: angles in a triangle add up to  $180^\circ$

b)

$$\angle CAQ = 180^\circ - 64^\circ - 50^\circ$$

$$\angle CAQ = 66^\circ$$

Reason: angles on a straight line add up to  $180^\circ$

(this matches  $\angle ABC$ , as the alternate segment theorem says)

|                             |
|-----------------------------|
| a) $66^\circ$ b) $66^\circ$ |
|-----------------------------|

6

A and B are points on a circle with centre O.

TA and TB are tangents to the circle from the point T.

Angle  $ATB = 48^\circ$ .

Work out the size of angle AOB. Give a reason for each stage of your working.

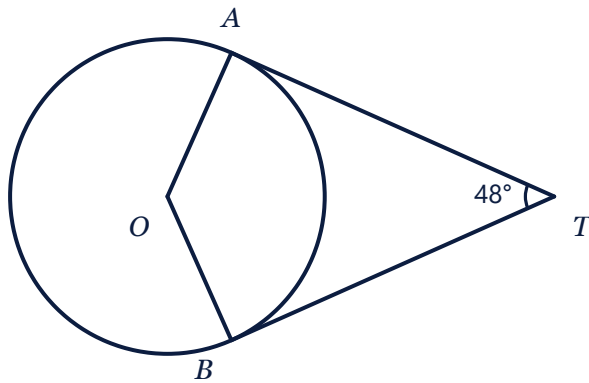


Diagram NOT accurately drawn

(3 marks)

Angle  $OAT = \text{angle } OBT = 90^\circ$  because a tangent to a circle is perpendicular to the radius at the point of contact

Angles in a quadrilateral add up to  $360^\circ$

$$\text{Angle } AOB = 360^\circ - 90^\circ - 90^\circ - 48^\circ = 132^\circ$$

132°

7

TP is a tangent to a circle with centre O. P is the point where the tangent touches the circle. Angle POT =  $58^\circ$ .

- Work out the size of angle PTO. Give a reason for each stage of your working.
- Kai says, "Angle PTO =  $122^\circ$  because the angles in a triangle add up to  $180^\circ$  and  $180^\circ - 58^\circ = 122^\circ$ ." Explain what Kai has done wrong.

(4 marks)

a)

$$OPT = 90^\circ$$

Reason: a tangent is perpendicular to the radius

$$PTO = 180^\circ - 90^\circ - 58^\circ$$

$$PTO = 32^\circ$$

Reason: angles in a triangle add up to  $180^\circ$

b)

$180^\circ - 58^\circ = 122^\circ$  is the  $90^\circ$  and the  $32^\circ$  added together

Kai forgot the right angle at P

|                                                   |
|---------------------------------------------------|
| $a) 32^\circ$ $b) \text{ forgot } OPT = 90^\circ$ |
|---------------------------------------------------|

8

A, B, C and D are points on a circle, in that order.

Angle ABD =  $29^\circ$  and angle DBC =  $46^\circ$ .

Work out the size of

- angle ACD,
- angle ADC.

Give a reason for each stage of your working.

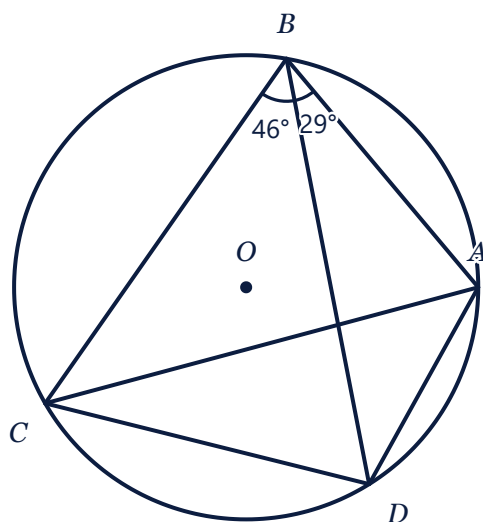


Diagram NOT accurately drawn

(4 marks)

a) Angle ACD = angle ABD =  $29^\circ$  because angles in the same segment, standing on chord AD, are equal

$$b) \text{Angle ABC} = 29^\circ + 46^\circ = 75^\circ$$

Opposite angles of a cyclic quadrilateral add up to  $180^\circ$

$$\text{Angle ADC} = 180^\circ - 75^\circ = 105^\circ$$

9

A, B and C are points on a circle with centre O.

C lies on the major arc AB.

Angle OAB =  $28^\circ$ .

Work out the size of angle ACB. Give a reason for each stage of your working.

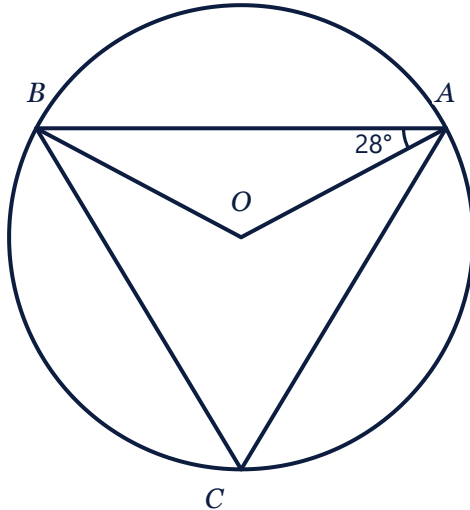


Diagram NOT accurately drawn

(4 marks)

$OA = OB$  because both are radii, so triangle  $OAB$  is isosceles and angle  $OBA =$  angle  $OAB = 28^\circ$

Angles in a triangle add up to  $180^\circ$ , so angle  $AOB = 180^\circ - 28^\circ - 28^\circ = 124^\circ$

The angle at the centre is twice the angle at the circumference standing on the same arc

Angle  $ACB = 124^\circ \div 2 = 62^\circ$

62

10

A and C are points on a circle with centre O.

PC is a tangent to the circle at C.

Angle PCA =  $58^\circ$ .

Work out the size of angle OAC. Give a reason for each stage of your working.

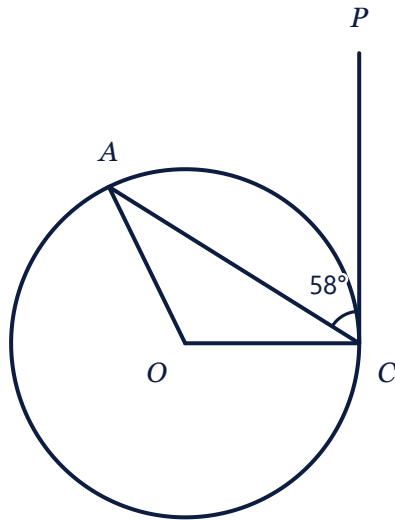


Diagram NOT accurately drawn

(4 marks)

Angle  $OCP = 90^\circ$  because a tangent to a circle is perpendicular to the radius at the point of contact

$$\text{Angle } OCA = 90^\circ - 58^\circ = 32^\circ$$

$OA = OC$  because both are radii, so triangle OAC is isosceles

$$\text{Angle } OAC = \text{angle } OCA = 32^\circ$$

32

11

A, B and C are points on a circle with centre O.

B lies on the minor arc AC.

Angle AOC =  $114^\circ$ .

Work out the size of

a. angle ABC,

b. angle OAC.

Give a reason for each stage of your working.

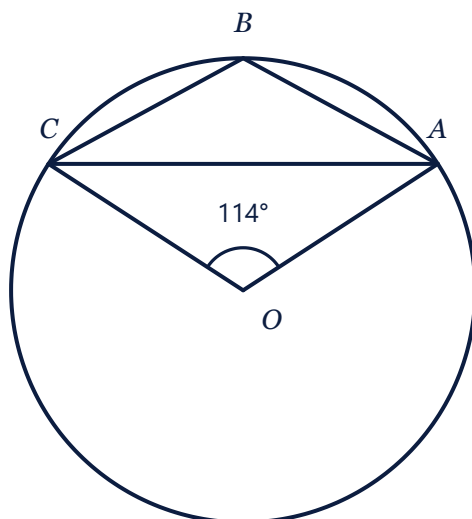


Diagram NOT accurately drawn

(4 marks)

a) The reflex angle AOC =  $360^\circ - 114^\circ = 246^\circ$  because angles around a point add up to  $360^\circ$

The angle at the centre is twice the angle at the circumference standing on the same arc. Angle ABC stands on the major arc, so it pairs with the reflex angle at O

$$\text{Angle ABC} = 246^\circ \div 2 = 123^\circ$$

b) OA = OC because both are radii, so triangle OAC is isosceles

$$\text{Angle OAC} = (180^\circ - 114^\circ) \div 2 = 33^\circ$$

12

AB is a diameter of a circle with centre O. C is a point on the circle. The tangent to the circle at C meets AB extended beyond B at the point T.

Angle CAB =  $25^\circ$ .

- Work out the size of angle COB.
- Work out the size of angle CTO.

Give a reason for each stage of your working.

(5 marks)

a)

$$\text{COB} = 2 \times 25^\circ$$

$$\text{COB} = 50^\circ$$

Reason: angle at the centre is twice the angle at the circumference

b)

$$\text{OCT} = 90^\circ$$

Reason: a tangent is perpendicular to the radius

$$\text{CTO} = 180^\circ - 90^\circ - 50^\circ$$

$$\text{CTO} = 40^\circ$$

Reason: angles in a triangle add up to  $180^\circ$

|                             |
|-----------------------------|
| a) $50^\circ$ b) $40^\circ$ |
|-----------------------------|