



- 1 Write $x^2 + 10x + 7$ in the form $(x + a)^2 + b$, where a and b are integers.

(2 marks)

$$= x^2 + 10x + 7$$

Half of 10 is 5

$$= (x + 5)^2 - 25 + 7$$

$$(x + 5)^2 - 18$$

- 2 Write $x^2 + 8x + 3$ in the form $(x + a)^2 + b$, where a and b are integers.

(2 marks)

Expanding $(x + a)^2$ gives $x^2 + 2ax + a^2$. To match $8x$, we need $2a = 8$, so $a = 4$

Halve the coefficient of x

$$(x + 4)^2 = x^2 + 8x + 16$$

Adjust the constant

$$x^2 + 8x + 3 = (x + 4)^2 - 16 + 3 = (x + 4)^2 - 13$$

Check by expanding

$$x^2 + 8x + 16 - 13 = x^2 + 8x + 3$$

$$(x + 4)^2 - 13$$

3

a. Write $x^2 - 5x + 1$ in the form $(x - a)^2 + b$.b. Hence write down the minimum value of $x^2 - 5x + 1$ and the value of x for which it occurs.

(3 marks)

a)

$$= x^2 - 5x + 1$$

Half of -5 is $-5/2$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 1$$

$$-\frac{25}{4} + 1 = -\frac{21}{4}$$

$$\boxed{\left(x - \frac{5}{2}\right)^2 - \frac{21}{4}}$$

b)

The square is smallest (zero) when $x = 5/2$

$$\boxed{\text{minimum} = -\frac{21}{4} \text{ when } x = \frac{5}{2}}$$

4

a. Write $x^2 - 6x + 11$ in the form $(x - a)^2 + b$, where a and b are integers.b. Hence find the coordinates of the turning point of the curve $y = x^2 - 6x + 11$

(3 marks)

$$a) (x - 3)^2 = x^2 - 6x + 9, \text{ so } x^2 - 6x + 11 = (x - 3)^2 + 2$$

a) Check by expanding

$$x^2 - 6x + 9 + 2 = x^2 - 6x + 11$$

b) The minimum of $(x - 3)^2 + 2$ is when $x = 3$, giving the turning point $(3, 2)$

5

The curve with equation $y = x^2 + bx + c$ has its turning point at $(3, -4)$.Find the values of b and c .

(3 marks)

Turning point $(3, -4)$ gives the completed square form

$$y = (x - 3)^2 - 4$$

Expand the bracket

$$y = x^2 - 6x + 9 - 4$$

$$y = x^2 - 6x + 5$$

$$\boxed{b = -6, c = 5}$$

6

Solve $x^2 - 5x + 1 = 0$ by completing the square.Give your answers in the form $p \pm \frac{\sqrt{q}}{2}$ where p and q are integers.

(4 marks)

$$x^2 - 5x + 1 = 0$$

Complete the square

$$\begin{aligned} (x - \frac{5}{2})^2 - \frac{21}{4} &= 0 \\ +\frac{21}{4} \quad (x - \frac{5}{2})^2 &= \frac{21}{4} \quad +\frac{21}{4} \\ x - \frac{5}{2} &= \pm \frac{\sqrt{21}}{2} \\ +\frac{5}{2} \quad x &= \frac{5}{2} \pm \frac{\sqrt{21}}{2} \quad +\frac{5}{2} \end{aligned}$$

$$x = \frac{5 \pm \sqrt{21}}{2}$$

7

By completing the square, solve

$$x^2 + 6x + 4 = 0$$

Give your answers in the form $a \pm \sqrt{b}$ where a and b are integers.

(3 marks)

Complete the square

$$x^2 + 6x + 4 = (x + 3)^2 - 9 + 4 = (x + 3)^2 - 5$$

$$\text{So } (x + 3)^2 = 5$$

Both $\sqrt{5}$ and $-\sqrt{5}$ square to 5, so $x + 3 = \pm\sqrt{5}$. The $\pm$ sign represents these two possibilities

$$x = -3 \pm \sqrt{5}$$

Check by expanding

$$(x + 3)^2 - 5 = x^2 + 6x + 9 - 5 = x^2 + 6x + 4$$

$$x = -3 + \sqrt{5} \text{ or } x = -3 - \sqrt{5}$$

8

a. Write $3 + 8x - x^2$ in the form $a - (x - b)^2$, where a and b are integers.

b. Hence write down the coordinates of the maximum point of the curve $y = 3 + 8x - x^2$

(4 marks)

a)

$$3 + 8x - x^2 = 3 - (x^2 - 8x)$$

Complete the square inside the bracket

$$= 3 - ((x - 4)^2 - 16)$$

$$= 3 - (x - 4)^2 + 16$$

$$\boxed{19 - (x - 4)^2}$$

b)

$(x - 4)^2$ is never negative, so the greatest value is 19, when $x = 4$

$$\boxed{\text{maximum point } (4, 19)}$$

9

a. Write $2x^2 + 12x + 5$ in the form $a(x + b)^2 + c$, where a , b and c are integers.

b. Hence write down the coordinates of the turning point of the curve $y = 2x^2 + 12x + 5$

(4 marks)

a) Take out the factor 2 from the x terms: $2(x^2 + 6x) + 5$

a) Complete the square inside

$$x^2 + 6x = (x + 3)^2 - 9$$

$$\text{a) So } 2x^2 + 12x + 5 = 2(x + 3)^2 - 18 + 5 = 2(x + 3)^2 - 13$$

a) Check by expanding

$$2(x^2 + 6x + 9) - 13 = 2x^2 + 12x + 5$$

b) A square is at least zero, and its coefficient 2 is positive. The smallest value is therefore -13, when $x + 3 = 0$: the turning point is $(-3, -13)$

10

a. Write $3x^2 - 12x + 5$ in the form $a(x - b)^2 + c$, where a , b and c are integers.

b. Hence solve $3x^2 - 12x + 5 = 0$

Give your answers in the form $p \pm \frac{\sqrt{q}}{r}$ where p , q and r are integers.

(5 marks)

a)

$$\begin{aligned} 3x^2 - 12x + 5 &= 3(x^2 - 4x) + 5 \\ &= 3((x - 2)^2 - 4) + 5 \\ &= 3(x - 2)^2 - 12 + 5 \end{aligned}$$

$$\boxed{3(x - 2)^2 - 7}$$

b)

$$\begin{array}{rcl} 3(x - 2)^2 - 7 & = & 0 \\ +7 & & +7 \\ 3(x - 2)^2 & = & 7 \\ \div 3 & & \div 3 \\ (x - 2)^2 & = & \frac{7}{3} \\ & & \sqrt{} \\ x - 2 & = & \pm \sqrt{\frac{7}{3}} \end{array}$$

Rationalise: $\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

$$\boxed{x = 2 \pm \frac{\sqrt{21}}{3}}$$

11 $f(x) = 2x^2 - 8x + 3$

a. Write $f(x)$ in the form $a(x - b)^2 + c$, where a , b and c are integers.

b. Hence solve $f(x) = 0$, giving your answers in the form $p \pm \frac{\sqrt{q}}{2}$

c. Priya says that the minimum point of the curve $y = f(x)$ is $(-2, -5)$.

Explain the mistake Priya has made and write down the correct minimum point.

(7 marks)

a) Take out the factor 2 from the x terms: $2(x^2 - 4x) + 3$

a) Complete the square

$$x^2 - 4x = (x - 2)^2 - 4$$

a) So $f(x) = 2(x - 2)^2 - 8 + 3 = 2(x - 2)^2 - 5$

a) Check by expanding

$$2(x^2 - 4x + 4) - 5 = 2x^2 - 8x + 3$$

b) Solve $2(x - 2)^2 - 5 = 0$: $(x - 2)^2 = \frac{5}{2}$

$$b) x - 2 = \pm \sqrt{\left(\frac{5}{2}\right)} = \pm \frac{\sqrt{10}}{2}, \text{ so } x = 2 \pm \frac{\sqrt{10}}{2}$$

b) Check with the formula

$$x = \frac{8 \pm \sqrt{(64 - 24)}}{4} = \frac{8 \pm \sqrt{40}}{4} = 2 \pm \frac{\sqrt{10}}{2}$$

c) Since 2 is positive and $(x - 2)^2$ is at least zero, the minimum occurs when $x - 2 = 0$. This means $x = 2$, not -2 . Priya copied the sign inside the bracket instead of finding the x -value that makes it zero

c) The correct minimum point is $(2, -5)$