



1 $f(x) = 3x - 4$ $g(x) = x^2$

a. Find $f(5)$

b. Find $gf(2)$

(3 marks)

$gf(x)$ means $g(f(x))$: apply f first, then put its output into g . The function nearest the input is done first; gf does not mean multiply g by f

a) $f(5) = 3(5) - 4 = 11$

b) $f(2) = 3(2) - 4 = 2$, then $g(2) = 2^2 = 4$

2 $f(x) = 2x + 5$

a. Find $f^{-1}(x)$

b. Find $f^{-1}(11)$

(3 marks)

The inverse function reverses f : undo $+5$ by subtracting 5, then undo $\times 2$ by dividing by 2. f^{-1} means inverse function, not $1/f$

a) Write $y = 2x + 5$, swap to make x the subject: $x = \frac{y - 5}{2}$, so

$f^{-1}(x) = \frac{x - 5}{2}$

b) $f^{-1}(11) = \frac{11 - 5}{2} = 3$. Check: $f(3) = 11$

3 $f(x) = \frac{x + 2}{3}$ $g(x) = x^2 - 1$

a. Find $fg(3)$

b. Find $f^{-1}(x)$

(4 marks)

a) $g(3) = 9 - 1 = 8$, then $f(8) = \frac{8 + 2}{3} = \frac{10}{3}$

b) Write $y = \frac{x + 2}{3}$, so $3y = x + 2$ and $x = 3y - 2$, giving $f^{-1}(x) = 3x - 2$

b) Check

$f(3x - 2) = \frac{3x - 2 + 2}{3} = x$

4 $f(x) = 2x - 1$ $g(x) = x^2 + 3$

a. Show that $gf(x) = 4x^2 - 4x + 4$

b. Hence find $gf(3)$

(4 marks)

a) $gf(x) = g(2x - 1) = (2x - 1)^2 + 3$

a) Expand

$(2x - 1)^2 = 4x^2 - 4x + 1$, so $gf(x) = 4x^2 - 4x + 4$ as required

b) $gf(3) = 4(9) - 4(3) + 4 = 36 - 12 + 4 = 28$

b) Check directly

$f(3) = 5$, $g(5) = 25 + 3 = 28$

5 $f(x) = x^2 + 2x$

Express $f(x + 3)$ in the form $x^2 + bx + c$ where b and c are integers.

(3 marks)

$f(x + 3) = (x + 3)^2 + 2(x + 3)$

Expand: $x^2 + 6x + 9 + 2x + 6$

Collect terms: $x^2 + 8x + 15$

$x^2 + 8x + 15$

6 $f(x) = 3x + 2$ $g(x) = x^2 - 4$

a. Find $f^{-1}(x)$

b. Solve $gf(x) = 0$

(5 marks)

a) Write $y = 3x + 2$, so $x = \frac{y - 2}{3}$ and $f^{-1}(x) = \frac{x - 2}{3}$

b) $gf(x) = (3x + 2)^2 - 4 = 9x^2 + 12x + 4 - 4 = 9x^2 + 12x$

b) Factorise

$3x(3x + 4) = 0$, so $x = 0$ or $x = -\frac{4}{3}$

b) Check $x = 0$: $f(0) = 2$, $g(2) = 0$. Check $x = -\frac{4}{3}$: $f(-\frac{4}{3}) = -2$, $g(-2) = 0$

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$$f(x) = 4x - 1 \quad g(x) = x^2 + 2$$

- Find $fg(-3)$
- Find $gf(x)$, giving your answer in the form $ax^2 + bx + c$
- Find $f^{-1}(x)$

(5 marks)

$$\begin{aligned} \text{a) } g(-3) &= 9 + 2 = 11, \text{ then } f(11) = 44 - 1 = 43 \\ \text{b) } gf(x) &= (4x - 1)^2 + 2 = 16x^2 - 8x + 1 + 2 = 16x^2 - 8x + 3 \\ \text{c) Write } y &= 4x - 1, \text{ so } x = \frac{y+1}{4} \text{ and } f^{-1}(x) = \frac{x+1}{4} \end{aligned}$$

8

$$f(x) = x^2 \quad g(x) = x + 2$$

- Find $fg(x)$, giving your answer in the form $x^2 + bx + c$
- Solve $fg(x) = gf(x)$

(5 marks)

$$\begin{aligned} \text{a) } fg(x) &= (x + 2)^2 = x^2 + 4x + 4 \\ \text{b) } gf(x) &= x^2 + 2 \\ \text{b) Set them equal} \\ x^2 + 4x + 4 &= x^2 + 2, \text{ so } 4x = -2 \text{ and } x = -\frac{1}{2} \\ \text{b) Check} \\ fg\left(-\frac{1}{2}\right) &= \left(\frac{3}{2}\right)^2 = \frac{9}{4} \text{ and } gf\left(-\frac{1}{2}\right) = \frac{1}{4} + 2 = \frac{9}{4} \end{aligned}$$

9

$$f(x) = \frac{2x+1}{x-3}, x \neq 3$$

a. Show that $f^{-1}(x) = \frac{3x+1}{x-2}$

b. $g(x) = x + 4$

Solve $fg(x) = 5$

(6 marks)

a) Write $y = \frac{2x+1}{x-3}$ and multiply up: $y(x-3) = 2x+1$

a) Expand and collect x terms

$$xy - 2x = 3y + 1, \text{ so } x(y-2) = 3y+1$$

a) Divide

$$x = \frac{3y+1}{y-2}, \text{ so } f^{-1}(x) = \frac{3x+1}{x-2} \text{ as required}$$

The inverse is defined for $x \neq 2$

f never outputs 2, since $(2x+1)/(x-3) = 2$ would imply $1 = -6$. In part (b), $x \neq -1$ because $g(x)$ must not equal 3

$$\begin{aligned} \text{b) } fg(x) &= f(x+4) = \\ &= \frac{2(x+4)+1}{(x+4)-3} = \\ &= \frac{2x+9}{x+1} \end{aligned}$$

$$\begin{aligned} \text{b) Solve } \frac{2x+9}{x+1} &= 5: 2x+9 = 5x+5, \text{ so } 3x = 4 \text{ and } x \\ &= \frac{4}{3} \end{aligned}$$

b) Check

$$g\left(\frac{4}{3}\right) = \frac{16}{3} \text{ and } f\left(\frac{16}{3}\right) = \frac{35}{3} \cdot \frac{7}{3} = 5$$

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$$f(x) = x + 2 \quad g(x) = x^2 - 3x$$

- Find $fg(1)$
- Find $f^{-1}(x)$
- Solve $gf(x) = 10$

(7 marks)

$$a) g(1) = 1 - 3 = -2, \text{ then } f(-2) = 0$$

$$b) y = x + 2 \text{ gives } x = y - 2, \text{ so } f^{-1}(x) = x - 2$$

$$c) gf(x) = (x + 2)^2 - 3(x + 2) = x^2 + 4x + 4 - 3x - 6 = x^2 + x - 2$$

$$c) \text{ Solve } x^2 + x - 2 = 10, \text{ so } x^2 + x - 12 = 0$$

c) Factorise

$$(x + 4)(x - 3) = 0, \text{ so } x = -4 \text{ or } x = 3$$

c) Check $x = 3$: $f(3) = 5$, $g(5) = 25 - 15 = 10$. Check $x = -4$: $f(-4) = -2$, $g(-2) = 4 - 6 = -2$.
 $6 = 10$

11

$$f(x) = \frac{1}{x-4} \quad \text{and} \quad g(x) = \sqrt{x+3}$$

- State the value of x that must be excluded from the domain of f .
- State the largest possible domain of g .
- State the range of g .
- $fg(x)$ is not defined for one value of x in the domain of g . Find this value.

(4 marks)

a)

$$x - 4 = 0 \text{ gives } x = 4$$

$$x \neq 4$$

b)

$$x + 3 \geq 0$$

$$x \geq -3$$

c)

A square root is never negative

$$g(x) \geq 0$$

d)

$$\begin{array}{rcl}
 fg(x) & = & \sqrt{x+3} - 4 \\
 \sqrt{x+3} - 4 & = & 0 \\
 +4 \quad \sqrt{x+3} & = & 4 \quad +4 \\
 x + 3 & = & 16
 \end{array}$$

$$x = 13$$

12

$$f(x) = 3x + 10 \text{ and } g(x) = \sin x^\circ$$

- Work out $fg(30)$.
- Work out $gf(20)$, correct to 3 decimal places.
- Ahmed solves $gf(x) = 0.5$ and writes " $3x + 10 = 30$, so $x = \frac{20}{3}$ is the only solution." Show that Ahmed is wrong by finding another solution between 40 and 50.

(5 marks)

a)

$$g(30) = \sin 30^\circ = 0.5$$

$$fg(30) = f(0.5) = 3 \times 0.5 + 10$$

11.5

b)

$$f(20) = 3 \times 20 + 10 = 70$$

$$gf(20) = \sin 70^\circ$$

0.940

c)

$$\sin 30^\circ = \sin 150^\circ = 0.5$$

$$3x + 10 = 150$$

-10

$$3x = 140$$

-10

$$x = \frac{140}{3} = 46.67$$

Another solution: $x = 46.67$

13

 $f(x) = x^2 - 6x + 11$ for all real x .

- Write $f(x)$ in the form $(x - a)^2 + b$.
- Hence state the range of f .
- Explain why f does not have an inverse function for all real x .

(4 marks)

a)

$$x^2 - 6x + 11 = (x - 3)^2 - 9 + 11$$

$$(x - 3)^2 + 2$$

b)

$$(x - 3)^2 \geq 0$$

$$f(x) \geq 2$$

c)

$$f(2) = 4 - 12 + 11 = 3$$

$$f(4) = 16 - 24 + 11 = 3$$

Two inputs give one output, so no inverse