Domain and range of a function

Worked solutions

GCSE Maths topic practice

This topic online

1 The function f is defined by $f : x \mapsto 3x - 7$

a. Work out $f(5)$.

b. Work out $f(-2)$.

(2 marks)

a)

$f(5) = 3 \times 5 - 7$

$= 15 - 7$

8

b)

$f(-2) = 3 \times (-2) - 7$

$= -6 - 7$

-13

2 $f(x) = \frac{5}{x - 6}$

State the value of x that cannot be included in the domain of f.

(1 mark)

The bottom cannot be 0

$x - 6 \neq 0$

$+6 \quad x \neq 6 \quad +6$

$x = 6$ is excluded

3 The function f has domain {1, 2, 3, 4} and is defined by $f : x \mapsto x^2 - 1$

List the members of the range of f.

(2 marks)

Put every input into f

$f(1) = 1 - 1 = 0$

$f(2) = 4 - 1 = 3$

$f(3) = 9 - 1 = 8$

$f(4) = 16 - 1 = 15$

Range = {0, 3, 8, 15}

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4 $h(x) = \sqrt{(x - 4)}$ and $g(x) = \frac{4}{2x + 3}$

a. State the largest possible domain of h.

b. State the value of x that must be excluded from the domain of g.

(3 marks)

a) Inside a square root must be $\geqslant 0$

$x - 4 \geqslant 0$

$x \geqslant 4$ (add 4 to both sides)

$\boxed{x \geqslant 4}$

b) The bottom cannot be 0

$2x + 3 = 0$

$-3 \quad -3$

$2x = -3$

$\div 2 \quad \div 2$

$x = -1.5$

$\boxed{x = -1.5 \text{ is excluded}}$

5 $f(x) = 3x - 2$ for $-1 \leq x \leq 4$

Find the range of f.

(2 marks)

Straight line, so use the end points

$f(-1) = 3 \times (-1) - 2 = -5$

$f(4) = 3 \times 4 - 2 = 10$

$\boxed{-5 \leqslant f(x) \leqslant 10}$

6 The function f is defined by $f : x \mapsto \frac{3}{x^2 - 16}$

Find the two values of x that must be excluded from the domain of f.

(2 marks)

The bottom cannot be 0

$x^2 - 16 = 0$

$+16 \quad +16$

$x^2 = 16$

$x = \pm 4$

$\boxed{x = 4 \text{ or } x = -4}$

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7 $f(x) = x^2$ for $-3 \leq x \leq 2$

Leon says that the range of f is $4 \leq f(x) \leq 9$.

Leon is wrong. Find the correct range of f. (2 marks)

0 is in the domain, so the smallest output is 0^2

$f(0) = 0$

$f(-3) = 9$

$f(2) = 4$

Biggest output is 9, from $x = -3$

$$0 \leq f(x) \leq 9$$

8 The function g is defined by $g : x \mapsto \frac{2}{x}$ for $x > 0$

a. Work out $g(0.4)$.

b. State the range of g. (3 marks)

a)

$$g(0.4) = \frac{2}{0.4}$$

$$5$$

b) $x > 0$, so $2 \div x$ is always positive

Big x gives $g(x)$ close to 0, but never 0

x close to 0 gives a huge $g(x)$

$$g(x) > 0$$

9 $f(x) = 2x^2 - 12x + 5$ for all real values of x.

a. Write $f(x)$ in the form $a(x - b)^2 + c$.

b. Hence find the range of f. (4 marks)

a) Take out the 2 first

$$2x^2 - 12x + 5 = 2(x^2 - 6x) + 5$$
$$= 2[(x - 3)^2 - 9] + 5$$
$$= 2(x - 3)^2 - 18 + 5$$

$$2(x - 3)^2 - 13$$

b) A square is never negative

$2(x - 3)^2 \geq 0$, so smallest $f(x)$ is -13 at $x = 3$

$$f(x) \geq -13$$

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10 The function f is defined by $f: x \mapsto x^2 + 3$ with domain $x \geq 0$

a. State the range of f.

b. Write down the domain of the inverse function f^{-1}.

c. Write down the range of f^{-1}.

(4 marks)

a) Smallest input is $x = 0$

$f(0) = 0 + 3 = 3$

$x^2 + 3$ only gets bigger as x increases

$\boxed{f(x) \geqslant 3}$

b) Domain of f^{-1} = range of f

$\boxed{x \geqslant 3}$

c) Range of f^{-1} = domain of f

$\boxed{f^{-1}(x) \geqslant 0}$

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