



1 Work out, without a calculator, giving each answer as a fraction.

a. $(3\frac{3}{8})^{\frac{2}{3}}$

b. $(6\frac{1}{4})^{-\frac{3}{2}}$

(3 marks)

a)

$$3\frac{3}{8} = \frac{27}{8}$$

$$(\frac{27}{8})^{2/3} = (\sqrt[3]{\frac{27}{8}})^2$$

$$= (\frac{3}{2})^2$$

$$\boxed{\frac{9}{4}}$$

b)

$$6\frac{1}{4} = \frac{25}{4}$$

$$(\frac{25}{4})^{-3/2} = (\frac{4}{25})^{3/2}$$

$$= (\sqrt{\frac{4}{25}})^3 = (\frac{2}{5})^3$$

$$\boxed{\frac{8}{125}}$$

2 Write down the value of $121^{\frac{1}{2}}$

(1 mark)

A power of $\frac{1}{2}$ means the positive square root

$$\sqrt{121} = 11$$

$$\boxed{11}$$

3

Priya says that $27^{\frac{1}{3}} = 9$ because $27 \div 3 = 9$

Explain why Priya is wrong.

(1 mark)

$27^{\frac{1}{3}}$ means the cube root of 27, not 27 divided by 3

Since $3 \times 3 \times 3 = 27$, the correct value is 3

4

Work out the value of $(\frac{36}{25})^{\frac{1}{2}}$

(2 marks)

A power of $\frac{1}{2}$ means the square root of the numerator and of the denominator

$\sqrt{36} = 6$ and $\sqrt{25} = 5$, so the answer is $\frac{6}{5}$

$$\boxed{\frac{6}{5}}$$

5

a. Work out the value of $8^{\frac{2}{3}}$

b. Work out the value of $25^{-\frac{1}{2}}$

c. Work out the value of 10^{-2}

(3 marks)

$$(a) 8^{\frac{2}{3}} = (\text{cube root of } 8)^2 = 2^2 = 4$$

$$(b) 25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

$$(c) 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

6

Simplify fully $(27a^9b^6)^{\frac{2}{3}}$

(3 marks)

Apply the power $\frac{2}{3}$ to each factor

$$27^{\frac{2}{3}} = (\text{cube root of } 27)^2 = 3^2 = 9$$

$$(a^9)^{\frac{2}{3}} = a^6 \text{ and } (b^6)^{\frac{2}{3}} = b^4$$

So the expression simplifies to $9a^6b^4$

$$\boxed{9a^6b^4}$$

7

- a. Work out the value of $64^{-\frac{2}{3}}$
 b. Work out the value of $(\frac{81}{16})^{-\frac{3}{4}}$

(4 marks)

$$(a) 64^{\frac{2}{3}} = (\text{cube root of } 64)^2 = 4^2 = 16, \text{ so } 64^{-\frac{2}{3}} = \frac{1}{16}$$

(b) The negative index turns the fraction over

$$(\frac{81}{16})^{-\frac{3}{4}} = (\frac{16}{81})^{\frac{3}{4}}$$

$$16^{\frac{3}{4}} = (\text{fourth root of } 16)^3 = 2^3 = 8 \text{ and } 81^{\frac{3}{4}} = 3^3 = 27$$

So the answer is $\frac{8}{27}$

8

$$9^x = \frac{1}{27}$$

Find the value of x.

(3 marks)

Write both sides as powers of 3

$$9^x = (3^2)^x = 3^{2x}$$

$$\frac{1}{27} = 3^{-3}$$

$$\text{So } 2x = -3, \text{ giving } x = -\frac{3}{2}$$

$$\boxed{-\frac{3}{2}}$$

9

a. $8^n \times 2^5 = 4^7$

Find the value of n.

- b. Write $x\sqrt{x}$ as a single power of x.

(4 marks)

(a) Write everything as powers of 2

$$8^n = 2^{3n} \text{ and } 4^7 = 2^{14}$$

$$\text{So } 2^{3n} \times 2^5 = 2^{3n+5} = 2^{14}$$

$$3n + 5 = 14, \text{ so } n = 3$$

$$(b) \sqrt{x} = x^{\frac{1}{2}}, \text{ so } x\sqrt{x} = x^1 \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$$

10

a. Work out the value of $27^{-\frac{2}{3}}$ b. $4^x = 32$ Find the value of x .

(4 marks)

$$(a) 27^{\frac{2}{3}} = (\text{cube root of } 27)^2 = 3^2 = 9, \text{ so } 27^{-\frac{2}{3}} = \frac{1}{9}$$

(b) Write both sides as powers of 2

$$4^x = 2^{2x} \text{ and } 32 = 2^5$$

$$\text{So } 2x = 5, \text{ giving } x = \frac{5}{2}$$

11

a. Work out the value of $(\sqrt{3})^6$ b. Work out the value of $(\frac{64}{125})^{-\frac{2}{3}}$ c. $32^n = \frac{1}{\sqrt{8}}$ Find the value of n .

(6 marks)

$$(a) (\sqrt{3})^6 = (3^{\frac{1}{2}})^6 = 3^3 = 27$$

(b) The negative index turns the fraction over: $(\frac{125}{64})^{\frac{2}{3}}$

$$125^{\frac{2}{3}} = 5^2 = 25 \text{ and } 64^{\frac{2}{3}} = 4^2 = 16, \text{ so the answer is } \frac{25}{16}$$

(c) Write both sides as powers of 2

$$32^n = 2^{5n}$$

$$\frac{1}{\sqrt{8}} = 8^{-\frac{1}{2}} = (2^3)^{-\frac{1}{2}} = 2^{-\frac{3}{2}}$$

$$\text{So } 5n = -\frac{3}{2}, \text{ giving } n = -\frac{3}{10}$$

12

 $(x + 2x^{-1})^2 = x^2 + a + bx^{-2}$ for all non-zero x .Find the value of a and the value of b .

(3 marks)

Expand the brackets

$$= (x + 2x^{-1})(x + 2x^{-1})$$

$$= x^2 + 2x \cdot x^{-1} + 2x^{-1} \cdot x + 4x^{-2}$$

$$= x^2 + 2 + 2 + 4x^{-2}$$

$$= x^2 + 4 + 4x^{-2}$$

$$a = 4, b = 4$$

13

$(27x^a)^{\frac{1}{3}} = bx^2$ for all positive x .

Find the value of a and the value of b .

(3 marks)

$$\begin{aligned}(27x^a)^{1/3} &= 27^{1/3} \times (x^a)^{1/3} \\ &= 3 \times x^{a/3}\end{aligned}$$

Compare with bx^2

$$\begin{array}{rcl} b & = & 3 \\ \frac{a}{3} & = & 2 \\ \times 3 & & \times 3 \\ a & = & 6 \end{array}$$

$a = 6, b = 3$