



- 1 The table shows the frequency densities for the lengths, l cm, of the worms found in a soil survey.

Length (l cm)	Frequency density
$0 < l \leq 5$	1.2
$5 < l \leq 15$	2.1
$15 < l \leq 20$	3.0
$20 < l \leq 40$	0.4

Work out the total number of worms found in the survey.

(3 marks)

In a histogram, bar area represents frequency. Width $\times$ height must therefore equal frequency: the height (frequency density) is frequency divided by class width

$$\text{Frequency} = \text{frequency density} \times \text{class width}$$

$$0 < l \leq 5$$

$$1.2 \times 5 = 6. \quad 5 < l \leq 15: 2.1 \times 10 = 21$$

$$15 < l \leq 20$$

$$3.0 \times 5 = 15. \quad 20 < l \leq 40: 0.4 \times 20 = 8$$

$$\text{Total} = 6 + 21 + 15 + 8 = 50$$

50

- 2 The table shows the times, t minutes, that 53 customers spent in a coffee shop.

Time (t minutes)	Frequency	Frequency density
$0 < t \leq 10$	18	
$10 < t \leq 15$	12	
$15 < t \leq 25$	15	
$25 < t \leq 45$	8	

Complete the frequency density column.

(3 marks)

$$\text{Frequency density} = \text{frequency} \div \text{class width}$$

$$18 \div 10 = 1.8 \text{ and } 12 \div 5 = 2.4$$

$$15 \div 10 = 1.5 \text{ and } 8 \div 20 = 0.4$$

3

The frequency densities for the times, t minutes, that 80 customers waited in a post office queue are shown in the table.

Time (t minutes)	Frequency density
$0 < t \leq 10$	2
$10 < t \leq 30$	1.5
$30 < t \leq 40$	3

Mia says, "The $0 < t \leq 10$ class must have the most customers because its bar is tall." Is Mia correct? You must show your working.

(2 marks)

Frequency = frequency density $\times$ class width

$$0-10: 2 \times 10 = 20$$

$$10-30: 1.5 \times 20 = 30$$

$$30-40: 3 \times 10 = 30$$

No - 0-10 has the fewest (20)

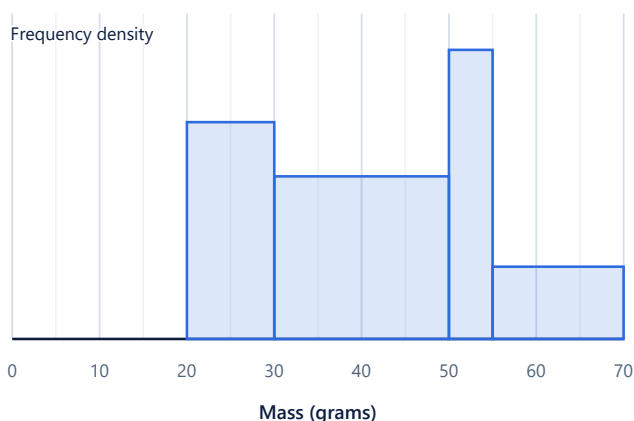
4

A histogram is drawn to show the masses, m grams, of some plums. The vertical axis has no scale, so the table records the height of each bar in centimetres.

Mass (m grams)	Bar height (cm)
$20 < m \leq 30$	6
$30 < m \leq 50$	4.5
$50 < m \leq 55$	8
$55 < m \leq 70$	2

There are 24 plums with masses in the class $20 < m \leq 30$.

- Work out the number of plums in the class $30 < m \leq 50$.
- Work out the total number of plums.



(4 marks)

For $20 < m \leq 30$

frequency 24, width 10, so frequency density = 2.4, meaning each cm of height represents 0.4 units of frequency density

$30 < m \leq 50$

frequency density = $4.5 \times 0.4 = 1.8$, frequency = $1.8 \times 20 = 36$

$50 < m \leq 55$

frequency density = $8 \times 0.4 = 3.2$, frequency = $3.2 \times 5 = 16$

$55 < m \leq 70$

frequency density = $2 \times 0.4 = 0.8$, frequency = $0.8 \times 15 = 12$

Total = $24 + 36 + 16 + 12 = 88$

5

The table shows information about the ages, a years, of the people at a village fete.

Age (a years)	Frequency	Frequency density
$0 < a \leq 10$	30	3
$10 < a \leq 30$	f	
$30 < a \leq 60$	45	1.5

- a. Write an expression, in terms of f , for the frequency density of the class $10 < a \leq 30$.
 b. The bar for this class is drawn 1.8 units high. Work out the value of f .

(3 marks)

a) class width = $30 - 10 = 20$

frequency density = $\frac{f}{20}$

b) height = frequency density

$\frac{f}{20} = 1.8$

$\times 20$ $f = 36$ $\times 20$

a) $\frac{f}{20}$ b) $f = 36$

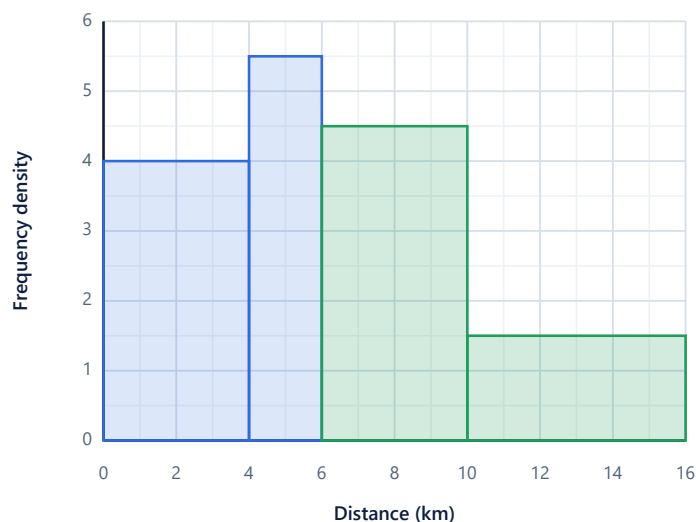
6

The table and histogram give information about the distances, d km, that some students travel to college.

Distance (d km)	Frequency
$0 < d \leq 4$	16
$4 < d \leq 6$	
$6 < d \leq 10$	18
$10 < d \leq 16$	

The bars for $0 < d \leq 4$ and $4 < d \leq 6$ are already drawn on the histogram. The bar for $4 < d \leq 6$ has frequency density 5.5.

- a. Use the histogram to work out the frequency for $4 < d \leq 6$.
 b. The frequency for $10 < d \leq 16$ is 9. Complete the histogram.



(4 marks)

Frequency = frequency density $\times$ width, so for $4 < d \leq 6$: $5.5 \times 2 = 11$

For $6 < d \leq 10$

frequency density = $18 \div 4 = 4.5$

For $10 < d \leq 16$

frequency density = $9 \div 6 = 1.5$

Draw the two missing bars at these heights

7

200 people were asked how long their journey to work takes. The table shows some of the results.

Time (t minutes)	Frequency	Frequency density
$0 < t \leq 10$	40	4
$10 < t \leq 30$	60	3
$30 < t \leq 40$		
$40 < t \leq 60$		

25% of the people take more than 30 minutes and up to 40 minutes. All the remaining people are in the last class.

- Work out the frequency density for $30 < t \leq 40$.
- Work out the frequency density for $40 < t \leq 60$.

(4 marks)

a)

$$25\% \text{ of } 200 = 50$$

$$fd = \frac{50}{10} = 5$$

b)

$$40 + 60 + 50 = 150$$

$$200 - 150 = 50$$

$$fd = \frac{50}{20} = 2.5$$

a) 5 b) 2.5

8

A histogram shows the times, t seconds, that 60 shoppers waited at a checkout. The table gives the frequency densities.

Time (t seconds)	Frequency density
$0 < t \leq 20$	0.6
$20 < t \leq 30$	2.0
$30 < t \leq 35$	3.2
$35 < t \leq 50$	0.8

- Show that the histogram represents 60 shoppers.
- One of the 60 shoppers is chosen at random. Work out an estimate for the probability that this shopper waited more than 40 seconds.

(4 marks)

For estimates using part of a class interval, assume the values are spread evenly within that interval.

The data do not give their exact positions within a class

Frequencies

$$0.6 \times 20 = 12, 2.0 \times 10 = 20, 3.2 \times 5 = 16, 0.8 \times 15 = 12. \text{ Total} = 60$$

Shoppers waiting more than 40 seconds lie in part of the last class: width $50 - 40 = 10$, so estimated frequency $= 0.8 \times 10 = 8$

$$\text{Estimated probability} = \frac{8}{60} = \frac{2}{15}$$

- 9 The table shows the frequency densities for the times, t seconds, that a number of pupils took to run 100 m. A pie chart is to be drawn to show the same information.

Time (t seconds)	Frequency density
$10 < t \leq 20$	0.8
$20 < t \leq 30$	2.4
$30 < t \leq 50$	1.1
$50 < t \leq 70$	0.3

Work out the angle of the sector of the pie chart for the class $30 < t \leq 50$.

(3 marks)

Frequency = $fd \times \text{class width}$

10-20: 8, 20-30: 24, 30-50: 22, 50-70: 6

Total = $8 + 24 + 22 + 6 = 60$

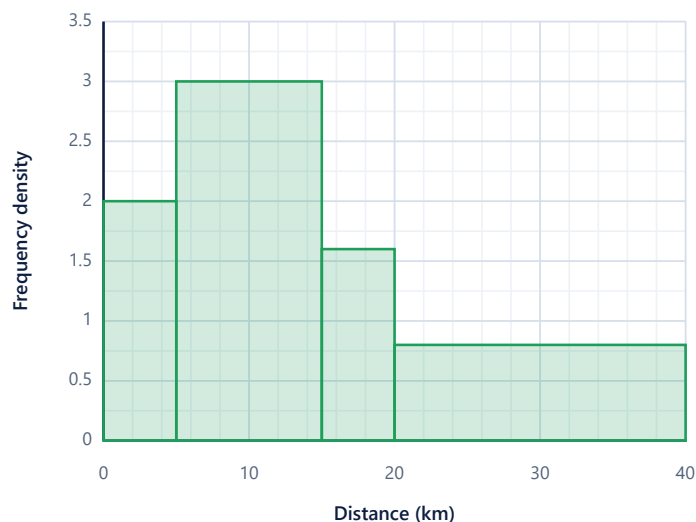
Angle = $\frac{22}{60} \times 360^\circ$

132°

- 10 The table shows the distances, d km, cycled by 64 members of a cycling club one Sunday.

Distance (d km)	Frequency
$0 < d \leq 5$	10
$5 < d \leq 15$	30
$15 < d \leq 20$	8
$20 < d \leq 40$	16

- On the grid, draw a histogram for this information.
- Explain why $5 < d \leq 15$ is the modal class even though $20 < d \leq 40$ has a wider bar.



(4 marks)

Frequency densities

$10 \div 5 = 2$, $30 \div 10 = 3$, $8 \div 5 = 1.6$, $16 \div 20 = 0.8$

Draw bars with these heights over the correct intervals

The modal class is the class with the highest frequency density; $5 < d \leq 15$ has frequency density 3, the tallest bar

11

The table shows the masses, m kg, of 50 lambs on a farm. A histogram is to be drawn.

Mass (m kg)	Frequency	Frequency density
$40 < m \leq 50$	6	
$50 < m \leq 60$	14	
$60 < m \leq 65$	15	
$65 < m \leq 80$	15	

a. Complete the frequency density column, to give the height of each bar.

b. Use the table to estimate the median mass of the lambs. Give your answer correct to 3 significant figures.

(5 marks)

a) $fd = \text{frequency} \div \text{class width}$

$$6 \div 10 = 0.6$$

$$14 \div 10 = 1.4$$

$$15 \div 5 = 3$$

$$15 \div 15 = 1$$

b) median is the 25th value

$$6 + 14 = 20$$

Need 5 more, in the class 60-65 (15 lambs)

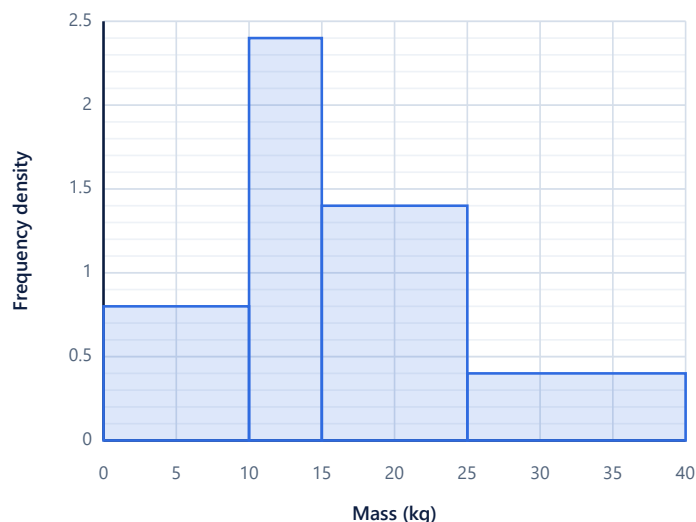
$$\text{Median} = 60 + \frac{5}{15} \times 5$$

$$\boxed{\text{a) } 0.6, 1.4, 3, 1 \text{ b) } 61.7 \text{ kg}}$$

12

The histogram shows the masses, m kg, of the 40 dogs seen at a vet's surgery one week.

- Use the histogram to complete a frequency table for the four classes.
- Write down the class interval that contains the lower quartile of the masses.
- Work out an estimate for the number of dogs with a mass greater than 20 kg.



(5 marks)

For estimates using part of a class interval, assume the values are spread evenly within that interval.

The data do not give their exact positions within a class

Frequencies from the histogram

$$0.8 \times 10 = 8, 2.4 \times 5 = 12, 1.4 \times 10 = 14, 0.4 \times 15 = 6. \text{ Total} = 40$$

The lower quartile is between the 10th and 11th values; the first class holds 8 dogs and the second reaches 20, so the lower quartile is in $10 < m \leq 15$

Greater than 20 kg

half of the $15 < m \leq 25$ class is 20 to 25, estimated frequency $1.4 \times 5 = 7$,

plus

all

6

dogs

in

25

$< m$

$\leq$

40

Estimate = 7 +

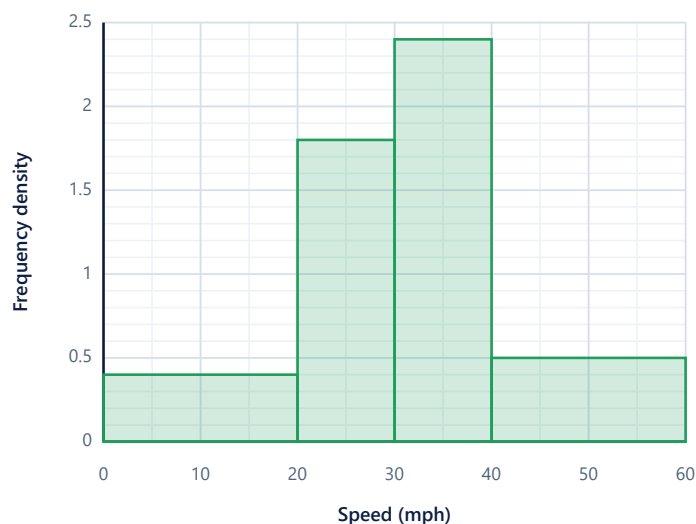
6 =

13

- 13 The table shows the speeds, s mph, of 60 vehicles passing a speed camera.

Speed (s mph)	Frequency
$0 < s \leq 20$	8
$20 < s \leq 30$	18
$30 < s \leq 40$	24
$40 < s \leq 60$	10

- a. On the grid, draw a histogram for this information.
b. Work out an estimate for the fraction of the vehicles travelling at more than 35 mph. Give your fraction in its simplest form.



(5 marks)

For estimates using part of a class interval, assume the values are spread evenly within that interval.

The data do not give their exact positions within a class

Frequency densities

$$8 \div 20 = 0.4, 18 \div 10 = 1.8, 24 \div 10 = 2.4, 10 \div 20 = 0.5$$

Draw bars with these heights

More than 35 mph

half of the $30 < s \leq 40$ class is estimated as $24 \div 2 = 12$ vehicles, plus all 10 vehicles above 40 mph

Estimate = 22 vehicles out of

$$60 = \frac{22}{60} = \frac{11}{30}$$

- 14 120 students sat a test. The table shows the frequency densities for their marks, m .

Mark (m)	Frequency density
$0 < m \leq 20$	0.5
$20 < m \leq 40$	1.5
$40 < m \leq 60$	2
$60 < m \leq 70$	2.5
$70 < m \leq 100$	0.5

The teacher awards a certificate to the top 10% of the students.

Estimate the lowest mark a student could get and still receive a certificate.

(4 marks)

Top 10% of 120 students

$$10\% \text{ of } 120 = 12$$

Top class 70-100: $0.5 \times 30 = 15$ students

The top 12 are inside this class

$$12 \text{ students} \div 0.5 = 24 \text{ marks}$$

$$100 - 24 = 76$$

76 marks

- 15 A gym has 200 members. A histogram shows the ages, a years, of the members. Some of the information is given in the table.

Age (a years)	Frequency	Frequency density
$0 < a \leq 10$	30	3
$10 < a \leq 25$		6
$25 < a \leq 45$		
$45 < a \leq 60$		1

a. Work out the frequency for the class $10 < a \leq 25$ and the frequency for the class $45 < a \leq 60$.

b. Work out the frequency and the frequency density for the class $25 < a \leq 45$.

c. Write down the class interval that contains the median age.

d. Work out an estimate for the number of members older than 50.

(6 marks)

For estimates using part of a class interval, assume the values are spread evenly within that interval.

The data do not give their exact positions within a class

$$10 < a \leq 25$$

$$\text{frequency} = 6 \times 15 = 90. \quad 45 < a \leq 60: \text{frequency} = 1 \times 15 = 15$$

$$25 < a \leq 45$$

$$\text{frequency} = 200 - 30 - 90 - 15 = 65, \text{ and frequency density} = 65 \div 20 = 3.25$$

Cumulative frequencies: 30, 120, 185, 200. The median is between the 100th and 101st values, which lie in $10 < a \leq 25$

Older than 50

$$\text{part of the last class from 50 to 60, estimated frequency} = 1 \times 10 = 10$$