



- 1 Here are the first five terms of a quadratic sequence.

2 7 14 23 34

Find an expression, in terms of n , for the n th term of this sequence.

(2 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences

5, 7, 9, 11. Second difference: 2, so the n^2 coefficient is 2 divided by 2 = 1

Subtract n^2 (1, 4, 9, 16, 25) from the sequence: 1, 3, 5, 7, 9, which has n th term

$2n - 1$

So the n th term is $n^2 + 2n - 1$

Check

$n = 1$ gives $1 + 2 - 1 = 2$; $n = 2$ gives $4 + 4 - 1 = 7$; $n = 3$ gives $9 + 6 - 1 = 14$; $n = 4$ gives $16 + 8 - 1 = 23$; $n = 5$ gives $25 + 10 - 1 = 34$

$$\boxed{n^2 + 2n - 1}$$

- 2 The n th term of a sequence is $n^2 + 4$.

- Write down the first three terms of the sequence.
- Is 85 a term of the sequence? You must show your working.
- Is 120 a term of the sequence? You must show your working.

(4 marks)

a)

$$n = 1: 1 + 4 = 5$$

$$n = 2: 4 + 4 = 8$$

$$n = 3: 9 + 4 = 13$$

b)

$$\begin{array}{rcl} n^2 + 4 & = & 85 \\ -4 & & \\ \hline n^2 & = & 81 \\ & & -4 \\ \hline n & = & 9 \end{array}$$

n is a whole number, so yes: the 9th term

c)

$$\begin{array}{rcl} n^2 + 4 & = & 120 \\ -4 & & \\ \hline n^2 & = & 116 \\ & & -4 \\ \hline \end{array}$$

$\sqrt{116} = 10.77... \text{ not a whole number}$

$\boxed{\text{No, 120 is not a term}}$

3 Here are the first five terms of a quadratic sequence.

5 11 19 29 41

Find an expression, in terms of n , for the n th term of this sequence.

(2 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences: 6, 8, 10, 12. Second difference: 2, so the n^2 coefficient is 1

Subtract n^2 (1, 4, 9, 16, 25) from the sequence: 4, 7, 10, 13, 16, which has n th term $3n + 1$

So the n th term is $n^2 + 3n + 1$

Check

$n = 1$ gives $1 + 3 + 1 = 5$; $n = 2$ gives $4 + 6 + 1 = 11$; $n = 3$ gives $9 + 9 + 1 = 19$; $n =$

4 gives $16 + 12 + 1 = 29$; $n = 5$ gives $25 + 15 + 1 = 41$

$$n^2 + 3n + 1$$

4 Here are the first five terms of a quadratic sequence.

1 10 25 46 73

Find an expression, in terms of n , for the n th term of this sequence.

(3 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences

9, 15, 21, 27. Second difference: 6, so the n^2 coefficient is 6 divided by 2 = 3

Subtract $3n^2$ (3, 12, 27, 48, 75) from the sequence: -2, -2, -2, -2, -2, a constant -2

So the n th term is $3n^2 - 2$

Check

$n = 1$ gives $3 - 2 = 1$; $n = 2$ gives $12 - 2 = 10$; $n = 3$ gives $27 - 2 = 25$; $n = 4$

gives $48 - 2 = 46$; $n = 5$ gives $75 - 2 = 73$

$$3n^2 - 2$$

5

A pattern of tiles is built in stages. Stage 1 uses 2 tiles, stage 2 uses 9 tiles, stage 3 uses 20 tiles and stage 4 uses 35 tiles. The number of tiles in each stage forms a quadratic sequence.

- Find an expression, in terms of n , for the number of tiles in stage n .
- Work out the number of tiles in stage 12.
- Mia says that one stage uses exactly 500 tiles. Is she correct? You must show your working.

(5 marks)

a)

Sequence: 2 9 20 35

1st diff: 7 11 15

2nd diff: 4 4

2nd difference 4, so $2n^2$ $2n^2$: 2 8 18 32Sequence - $2n^2$: 0 1 2 3This is $n - 1$

$$2n^2 + n - 1$$

b)

$$2 \times 12^2 + 12 - 1 = 288 + 11$$

$$299 \text{ tiles}$$

c)

$$n = 15: 2 \times 225 + 15 - 1 = 464$$

$$n = 16: 2 \times 256 + 16 - 1 = 527$$

500 is between 464 and 527

No, Mia is wrong

6

Here are the first five terms of a quadratic sequence.

6 15 28 45 66

Find an expression, in terms of n , for the n th term of this sequence.

(3 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences: 9, 13, 17, 21. Second difference: 4, so the n^2 coefficient is 2

Subtract $2n^2$ (2, 8, 18, 32, 50) from the sequence: 4, 7, 10, 13, 16, which has n th term $3n + 1$

So the n th term is $2n^2 + 3n + 1$

Check

$$n = 1 \text{ gives } 2 + 3 + 1 = 6; n = 2 \text{ gives } 8 + 6 + 1 = 15; n = 3 \text{ gives } 18 + 9 + 1 = 28; n = 4 \text{ gives } 32 + 12 + 1 = 45; n = 5 \text{ gives } 50 + 15 + 1 = 66$$

$$2n^2 + 3n + 1$$

7 Here are the first five terms of a quadratic sequence.

0 7 18 33 52

Find an expression, in terms of n , for the n th term of this sequence.

(3 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences: 7, 11, 15, 19. Second difference: 4, so the n^2 coefficient is 2

Subtract $2n^2$ (2, 8, 18, 32, 50) from the sequence: -2, -1, 0, 1, 2, which has n th term $n - 3$

So the n th term is $2n^2 + n - 3$

Check

$n = 1$ gives $2 + 1 - 3 = 0$; $n = 2$ gives $8 + 2 - 3 = 7$; $n = 3$ gives $18 + 3 - 3 = 18$; $n = 4$ gives $32 + 4 - 3 = 33$; $n = 5$ gives $50 + 5 - 3 = 52$

$$\boxed{2n^2 + n - 3}$$

8 Here are the first four terms of a quadratic sequence.

30 27 20 9

a. Find an expression, in terms of n , for the n th term.

b. Work out the value of the first term in the sequence that is negative.

(4 marks)

a)

Sequence: 30 27 20 9

1st diff: -3 -7 -11

2nd diff: -4 -4

2nd difference -4, so $-2n^2$

$-2n^2$: -2 -8 -18 -32

Sequence - $(-2n^2)$: 32 35 38 41

This is $3n + 29$

$$\boxed{-2n^2 + 3n + 29}$$

b)

$n = 4$: $-32 + 12 + 29 = 9$

$n = 5$: $-50 + 15 + 29 = -6$

$$\boxed{-6 \text{ (the 5th term)}}$$

9

Here are the first five terms of a quadratic sequence.

4 13 26 43 64

- Find an expression, in terms of n , for the n th term of this sequence.
- Hence find the 10th term of the sequence.

(3 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

a) First differences: 9, 13, 17, 21. Second difference: 4, so the n^2 coefficient is 2

Subtract $2n^2$ (2, 8, 18, 32, 50) from the sequence: 2, 5, 8, 11, 14, which has n th term $3n - 1$

So the n th term is $2n^2 + 3n - 1$

Check

$$n = 1 \text{ gives } 2 + 3 - 1 = 4; n = 2 \text{ gives } 8 + 6 - 1 = 13; n = 3 \text{ gives } 18 + 9 -$$

$$1 = 26; n = 4 \text{ gives } 32 + 12 - 1 = 43; n = 5 \text{ gives } 50 + 15 - 1 = 64$$

$$b) \text{ When } n = 10: 2(100) + 30 - 1 = 200 + 30 - 1 = 229$$

10

The first term of a quadratic sequence is 2. The differences between consecutive terms are 5, 9, 13, 17, ... Find an expression, in terms of n , for the n th term of the sequence.

(3 marks)

Build the sequence from the differences

2 7 16 29

1st diff: 5 9 13

2nd diff: 4 4

2nd difference 4, so $2n^2$

$2n^2$: 2 8 18 32

Sequence - $2n^2$: 0 -1 -2 -3

This is $-n + 1$

$$2n^2 - n + 1$$

11 Here are the first five terms of a quadratic sequence.

20 38 52 62 68

Find an expression, in terms of n , for the n th term of this sequence.

(3 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences: 18, 14, 10, 6. Second difference: -4, so the n^2 coefficient is -2

Subtract $-2n^2$ (-2, -8, -18, -32, -50) from the sequence: 22, 46, 70, 94, 118, which has n th term $24n - 2$

So the n th term is $-2n^2 + 24n - 2$

Check

$n = 1$ gives $-2 + 24 - 2 = 20$; $n = 2$ gives $-8 + 48 - 2 = 38$; $n = 3$ gives $-18 + 72 - 2 = 52$; $n = 4$ gives $-32 + 96 - 2 = 62$; $n = 5$ gives $-50 + 120 - 2 = 68$

$$\boxed{-2n^2 + 24n - 2}$$

12 Here are the first five terms of a quadratic sequence.

3 8 14 21 29

Find an expression, in terms of n , for the n th term of this sequence.

(3 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences: 5, 6, 7, 8. Second difference: 1, so the n^2 coefficient is $\frac{1}{2}$

Subtract $\frac{n^2}{2}$ (0.5, 2, 4.5, 8, 12.5) from the sequence: 2.5, 6, 9.5, 13, 16.5, an arithmetic sequence with n th term $3.5n - 1$

So the n th term is $0.5n^2 + 3.5n - 1$, which can be written as $\frac{n^2 + 7n - 2}{2}$

Check

$n = 1$ gives $\frac{1 + 7 - 2}{2} = 3$; $n = 2$ gives $\frac{4 + 14 - 2}{2} = 8$; $n = 3$ gives $\frac{9 + 21 - 2}{2} =$

14; $n = 4$ gives $\frac{16 + 28 - 2}{2} = 21$; $n = 5$ gives $\frac{25 + 35 - 2}{2} = 29$

$$\boxed{\frac{n^2 + 7n - 2}{2}}$$

13

The n th term of a sequence is $2n^2 + 5n$.Show that the difference between the $(n + 1)$ th term and the n th term is $4n + 7$.

(3 marks)

Replace n by $(n + 1)$

$$\begin{aligned}
 (n + 1)\text{th term} &= 2(n + 1)^2 + 5(n + 1) \\
 &= 2n^2 + 4n + 2 + 5n + 5 \\
 &= 2n^2 + 9n + 7
 \end{aligned}$$

Subtract the n th term

$$= 2n^2 + 9n + 7 - (2n^2 + 5n)$$

$4n + 7$

14

The n th term of a quadratic sequence is $an^2 + bn$, where a and b are constants.

The 2nd term of the sequence is 10.

The 4th term of the sequence is 36.

a. Find an expression, in terms of n , for the n th term of the sequence.b. Find the value of n for which the n th term of the sequence is 105.

(6 marks)

a) $n = 2$ gives $4a + 2b = 10$, so $2a + b = 5$. $n = 4$ gives $16a + 4b = 36$, so $4a + b = 9$

Subtracting

$$2a = 4, \text{ so } a = 2, \text{ and then } b = 5 - 4 = 1$$

The n th term is $2n^2 + n$

Check

$$2\text{nd term} = 8 + 2 = 10; 4\text{th term} = 32 + 4 = 36$$

$$b) \text{ Solve } 2n^2 + n = 105, \text{ so } 2n^2 + n - 105 = 0$$

Factorise

$$(2n + 15)(n - 7) = 2n^2 - 14n + 15n - 105 = 2n^2 + n - 105, \text{ so } n = 7 \text{ or } n = -\frac{15}{2}$$

 n must be a positive integer, so $n = 7$

Check

$$2(49) + 7 = 98 + 7 = 105$$

15

Here are the first five terms of a quadratic sequence.

7 16 29 46 67

- Find an expression, in terms of n , for the n th term of this sequence.
- Show that 500 is not a term of this sequence.

(6 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

a) First differences: 9, 13, 17, 21. Second difference: 4, so the n^2 coefficient is 2

Subtract $2n^2$ (2, 8, 18, 32, 50) from the sequence: 5, 8, 11, 14, 17, which has n th term $3n + 2$

So the n th term is $2n^2 + 3n + 2$

Check

$n = 1$ gives $2 + 3 + 2 = 7$; $n = 2$ gives $8 + 6 + 2 = 16$; $n = 3$ gives $18 + 9 + 2 = 29$;

$n = 4$ gives $32 + 12 + 2 = 46$; $n = 5$ gives $50 + 15 + 2 = 67$

b) If 500 were a term, then $2n^2 + 3n + 2 = 500$, so $2n^2 + 3n - 498 = 0$

The discriminant (the quantity $b^2 - 4ac$ under the square root in the quadratic formula) is $3^2 + 4 \times 2 \times 498 = 9 + 3984 = 3993$

3993 is not a square number ($63^2 = 3969$ and $64^2 = 4096$), so n is not a whole number

Alternatively

the 15th term is $2(225) + 45 + 2 = 497$ and the 16th term is $2(256) + 48 + 2 = 562$, and the sequence is increasing, so it skips 500

Either way, 500 is not a term of the sequence