



1 For each pair of lines, decide whether they are perpendicular. You must show your working.

a. $y = 4x + 1$ and $4y + x = 8$

b. $y = 2x + 5$ and $2y = x - 3$

(3 marks)

a)

$$m_1 = 4$$

$$4y = -x + 8$$

$$y = -\frac{1}{4}x + 2$$

$$m_2 = -\frac{1}{4}$$

$$4 \times \left(-\frac{1}{4}\right) = -1$$

Yes, perpendicular

b)

$$m_1 = 2$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

$$m_2 = \frac{1}{2}$$

$$2 \times \frac{1}{2} = 1$$

No, product is not -1

- 2 The line $y = ax + 5$ is perpendicular to the line $by = 3x - 2$.

Find b in terms of a .

(3 marks)

$$m_1 = a$$

$$y = \frac{3}{b}x - \frac{2}{b}$$

$$m_2 = \frac{3}{b}$$

Perpendicular: product of gradients = -1

$$a \times \frac{3}{b} = -1$$

$$\begin{array}{ccc} \times b & & \times b \\ 3a & = & -b \end{array}$$

$$b = -3a$$

- 3 The line l passes through the points $(-1, 2)$ and $(3, 10)$.

Find the equation of the line perpendicular to l that passes through the point $(4, 5)$. Give your answer in the form $y = mx + c$.

(3 marks)

$$\text{Change in } y: 10 - 2 = 8$$

$$\text{Change in } x: 3 - (-1) = 4$$

$$m_l = \frac{8}{4} = 2$$

$$m_{\perp} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + c$$

Substitute $(4, 5)$

$$5 = -\frac{1}{2}(4) + c$$

$$5 = -2 + c$$

$$\begin{array}{ccc} +2 & & +2 \\ c & = & 7 \end{array}$$

$$y = -\frac{1}{2}x + 7$$

4

A straight line has equation $y = 4x + 3$

Write down the gradient of a line perpendicular to this line.

(1 mark)

Perpendicular gradients multiply to give -1

The gradient of the given line is 4

The perpendicular gradient is $-\frac{1}{4}$, since $4 \times (-\frac{1}{4}) = -1$

$$\boxed{-\frac{1}{4}}$$

5

The line L is perpendicular to the line with equation $y = 2x - 3$

L passes through the point (4, 1).

Find an equation of L.

(3 marks)

The gradient of $y = 2x - 3$ is 2, so a perpendicular line has gradient $-\frac{1}{2}$ (the negative reciprocal, since 2 times $-\frac{1}{2} = -1$)

A line with gradient m through (a, b) satisfies $y - b = m(x - a)$: vertical change equals gradient $\times$ horizontal change. Here $(a, b) = (4, 1)$

Using $y - 1 = -\frac{1}{2}(x - 4)$:

$$y - 1 = -\frac{x}{2} + 2, \text{ so } y = -\frac{x}{2} + 3$$

Check

at $x = 4$, $y = -2 + 3 = 1$, so the line passes through (4, 1)

$$\boxed{y = -\frac{x}{2} + 3}$$

6

Find the equation of the straight line that is perpendicular to $y = 2x - 3$ and passes through the point (4, 1).

(3 marks)

The gradient of $y = 2x - 3$ is 2, so the perpendicular gradient is $-\frac{1}{2}$, since $2 \times (-\frac{1}{2}) = -1$

The new line has the form $y = -\frac{x}{2} + c$

Substitute (4, 1)

$$1 = -2 + c, \text{ so } c = 3$$

The equation is $y = -\frac{x}{2} + 3$

$$\boxed{y = -\frac{x}{2} + 3}$$

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The straight line L has equation $y = -\frac{x}{3} + 5$

Find the equation of the straight line that is perpendicular to L and passes through the point (2, -1).

(3 marks)

The gradient of L is $-\frac{1}{3}$, so the perpendicular gradient is 3,

since $-\frac{1}{3} \times 3 = -1$

The new line has the form $y = 3x + c$

Substitute (2, -1)

$$-1 = 6 + c, \text{ so } c = -7$$

The equation is $y = 3x - 7$

$$\boxed{y = 3x - 7}$$

8

The straight line L_1 has equation $y = px + 4$, where p is a constant.

The straight line L_2 has equation $2x + 6y = 9$

L_1 is perpendicular to L_2

Find the value of p .

(3 marks)

$$\text{Rearrange } 2x + 6y = 9: 6y = -2x + 9, \text{ so } y = -\frac{x}{3} + \frac{3}{2}$$

The gradient of L_2 is $-\frac{1}{3}$

Perpendicular gradients multiply to -1 , so $p \times (-\frac{1}{3}) = -1$

$$p = 3. \text{ Check: } 3 \times (-\frac{1}{3}) = -1, \text{ correct}$$

$$p = 3$$

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The straight line L_1 has equation $y = 2x + 6$

L_1 crosses the x -axis at the point A and crosses the y -axis at the point B.

The straight line L_2 is perpendicular to L_1 and passes through B.

L_2 crosses the x -axis at the point C.

Work out the area of triangle ABC.

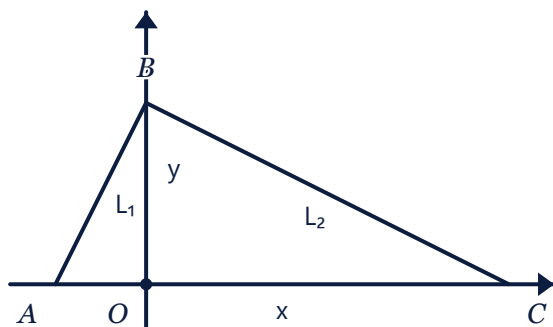


Diagram NOT accurately drawn

(5 marks)

A is where $y = 2x + 6$ meets $y = 0$: $2x + 6 = 0$, so A is $(-3, 0)$. B is $(0, 6)$

L_2 has gradient $-\frac{1}{2}$, since $2 \times (-\frac{1}{2}) = -1$, and passes through B, so L_2 is $y = -\frac{x}{2} + 6$

C is where $y = -\frac{x}{2} + 6$ meets $y = 0$: $\frac{x}{2} = 6$, so C is $(12, 0)$

Triangle ABC has base $AC = 12 - (-3) = 15$ on the x -axis and height 6 (the y -coordinate of B)

$$\text{Area} = \frac{1}{2} \times 15 \times 6 = 45$$

45

10

A is the point (0, 2) and B is the point (8, 6).

The point M lies on AB so that $AM : MB = 1 : 3$

Find the equation of the straight line that passes through M and is perpendicular to AB.

(5 marks)

AM

MB = 1 : 3 means M is $\frac{1}{4}$ of the way from A to B

$$M = (0 + \frac{1}{4} \times 8, 2 + \frac{1}{4} \times 4) = (2, 3)$$

The gradient of AB is $\frac{6-2}{8-0} = \frac{1}{2}$, so the perpendicular gradient is -2, since $\frac{1}{2} \times (-2) = -1$

The line has the form $y = -2x + c$. Substitute (2, 3): $3 = -4 + c$, so $c = 7$

The equation is $y = -2x + 7$

$$\boxed{y = -2x + 7}$$

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A is the point (-1, 3) and B is the point (5, 7).

Find the equation of the perpendicular bisector of AB. Give your answer in the form $y = mx + c$.

(4 marks)

Midpoint

$$(\frac{-1+5}{2}, \frac{3+7}{2}) = (2, 5)$$

Gradient of AB

Change in y: $7 - 3 = 4$, change in x: $5 - (-1) = 6$

$$m = \frac{4}{6} = \frac{2}{3}$$

$$m_{\perp} = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + c$$

$$5 = -\frac{3}{2}(2) + c$$

$$5 = -3 + c$$

$$\begin{array}{ccc} +3 & & +3 \\ c & = & 8 \end{array}$$

$$\boxed{y = -\frac{3}{2}x + 8}$$

12

ABCD is a rectangle, with the vertices in order. A is (1, 2), B is (5, 4) and C is (7, 0).

- Show that angle $ABC = 90^\circ$.
- Work out the coordinates of D.

(4 marks)

a)

$$m_{AB} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{-4}{2} = -2$$

$$\frac{1}{2} \times (-2) = -1$$

$$AB \perp BC, \text{ so angle } ABC = 90^\circ$$

b) D is found from A by the same move as B to C

B to C: 2 right, 4 down

$$D = (1 + 2, 2 - 4)$$

$$D = (3, -2)$$

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Line L_1 passes through (0, 1) and (1, $k + 1$).

Line L_2 passes through (0, 0) and (1, $k - 4$).

L_1 and L_2 are perpendicular.

Show that $k^2 - 4k + 1 = 0$ and hence find the possible values of k , giving your answers as surds.

(4 marks)

$$m_1 = k$$

$$m_2 = k - 4$$

$$k(k - 4) = -1$$

$$k^2 - 4k + 1 = 0$$

Quadratic formula, $a = 1$, $b = -4$, $c = 1$

$$k = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$k = 2 + \sqrt{3} \text{ or } k = 2 - \sqrt{3}$$