Proportion with square roots

Worked solutions

GCSE Maths topic practice

This topic online

1 y is directly proportional to the square root of x.

When $x = 9$, $y = 12$.

Find a formula for y in terms of x. (2 marks)

$y = k\sqrt{x}$

Put in $x = 9$, $y = 12$

$12 = k\sqrt{9} = 3k$

$\div 3$ $\quad k = 4$ $\quad \div 3$

$y = 4\sqrt{x}$

2 y is directly proportional to $\sqrt{x}$.

When $x = 25$, $y = 10$.

a. Find a formula for y in terms of x.

b. Find the value of y when $x = 64$.

c. Find the value of x when $y = 30$. (4 marks)

a)

$y = k\sqrt{x}$

$10 = k\sqrt{25} = 5k$

$\div 5$ $\quad k = 2$ $\quad \div 5$

$y = 2\sqrt{x}$

b)

$y = 2\sqrt{64} = 2 \times 8$

$y = 16$

c)

$30 = 2\sqrt{x}$

$\div 2$ $\quad 15 = \sqrt{x}$ $\quad \div 2$

Square both sides

$x = 15^2$

$x = 225$

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3 Here are descriptions of four sketch graphs. Each is drawn for $x > 0$.

Graph A: a straight line through the origin, sloping upwards.

Graph B: a curve starting at the origin that rises steeply at first and then flattens out, still rising, as x increases.

Graph C: a curve starting at the origin that rises slowly at first and then gets steeper and steeper as x increases.

Graph D: a curve that falls as x increases, getting closer and closer to the x-axis but never touching either axis.

a. Write down the letter of the graph that could be $y = k\sqrt{x}$, where $k > 0$.

b. Write down the letter of the graph that could be $y = \frac{k}{\sqrt{x}}$, where $k > 0$.

(2 marks)

a)

$\sqrt{0} = 0$ so it goes through the origin

$\sqrt{x}$ grows more and more slowly, so the curve flattens

Graph B

b)

Bigger x gives a bigger $\sqrt{x}$, so $\frac{k}{\sqrt{x}}$ gets smaller

Never reaches 0 and cannot use $x = 0$

Graph D

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4 y is inversely proportional to the square root of x.

When $x = 16$, $y = 5$.

a. Find a formula for y in terms of x.

b. Find the value of y when $x = 100$.

c. Find the value of x when $y = 8$.

(4 marks)

a)

$$y = \frac{k}{\sqrt{x}}$$

$$5 = \frac{k}{\sqrt{16}} = \frac{k}{4}$$

$\times 4$ $\quad k = 20$ $\quad \times 4$

$$y = \frac{20}{\sqrt{x}}$$

b)

$$y = \frac{20}{\sqrt{100}} = \frac{20}{10}$$

$$y = 2$$

c)

$$8 = \frac{20}{\sqrt{x}}$$

$\times \sqrt{x}$ $\quad 8\sqrt{x} = 20$ $\quad \times \sqrt{x}$

$\div 8$ $\quad \sqrt{x} = 2.5$ $\quad \div 8$

Square both sides

$$x = 2.5^2$$

$$x = 6.25$$

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5 The time, T seconds, for one swing of a pendulum is directly proportional to the square root of its length, L cm.

A pendulum of length 100 cm takes 2 seconds for one swing.

a. Find a formula for T in terms of L.

b. Find the time for one swing of a pendulum of length 225 cm.

c. Priya wants a pendulum that takes 2.4 seconds for one swing. Find, in cm, the length she should use.

(4 marks)

a)

$T = k\sqrt{L}$

$2 = k\sqrt{100} = 10k$

$\div 10$ $\quad k = 0.2$ $\quad \div 10$

$T = 0.2\sqrt{L}$

b)

$T = 0.2\sqrt{225} = 0.2 \times 15$

3 seconds

c)

$2.4 = 0.2\sqrt{L}$

$\div 0.2$ $\quad 12 = \sqrt{L}$ $\quad \div 0.2$

$L = 12^2$

$L = 144$ cm

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6 y is directly proportional to the square root of x.

Here is a table of values for x and y. Three of the values are missing.

x	4	25	(b)	144
y	6	(a)	24	(c)

Complete the table by finding the missing values.

a. the value of y when $x = 25$

b. the value of x when $y = 24$

c. the value of y when $x = 144$

(3 marks)

Find k from the full pair $x = 4$, $y = 6$

$y = k\sqrt{x}$

$6 = k\sqrt{4} = 2k$

$\div 2$ $\quad k = 3$ $\quad \div 2$

$y = 3\sqrt{x}$

a)

$y = 3\sqrt{25} = 3 \times 5$

$\boxed{y = 15}$

b)

$24 = 3\sqrt{x}$

$\div 3$ $\quad 8 = \sqrt{x}$ $\quad \div 3$

Square both sides

$x = 8^2$

$\boxed{x = 64}$

c)

$y = 3\sqrt{144} = 3 \times 12$

$\boxed{y = 36}$

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7 The speed, v metres per second, of a wave in shallow water is directly proportional to the square root of the depth of the water, d metres.

When the depth is 6.25 metres, the speed of the wave is 7.5 metres per second.

a. Find a formula for v in terms of d.

b. Find, in metres, the depth of the water when the speed of the wave is 10.5 metres per second. (3 marks)

a)

$v = k\sqrt{d}$

$7.5 = k\sqrt{6.25} = 2.5k$

$\div 2.5$ $\quad k = 3$ $\quad \div 2.5$

$v = 3\sqrt{d}$

b)

$10.5 = 3\sqrt{d}$

$\div 3$ $\quad 3.5 = \sqrt{d}$ $\quad \div 3$

$d = 3.5^2$

$d = 12.25$ m

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8 y is directly proportional to the square root of x.

a. x is multiplied by 4. Write down the number that y is multiplied by.

b. x is increased by 44%. Work out the percentage increase in y.

w is inversely proportional to the square root of x.

c. x is decreased by 36%. Work out the percentage increase in w.

(4 marks)

a)

$y = k\sqrt{x}$

new $y = k\sqrt{(4x)} = k \times 2\sqrt{x}$

y is multiplied by 2

b)

Up 44% means $\times 1.44$

new $y = k\sqrt{(1.44x)} = 1.2 \times k\sqrt{x}$

$\times 1.2$ means up 20%

20% increase

c)

Down 36% means $\times 0.64$

new $w = \frac{k}{\sqrt{(0.64x)}} = \frac{k}{0.8\sqrt{x}}$

$\frac{1}{0.8} = 1.25$

25% increase

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9 Here is a table of values for x and y.

x	4	16	100
y	10	5	2

Exactly one of these statements is true.

A: y is directly proportional to x.

B: y is directly proportional to $\sqrt{x}$.

C: y is inversely proportional to x.

D: y is inversely proportional to $\sqrt{x}$.

E: y is inversely proportional to x^2.

a. Write down the letter of the true statement.

b. Find a formula for y in terms of x.

c. Find the value of y when $x = 400$.

(4 marks)

a)

y goes down as x goes up, so inverse

Test $y \times x$: 40, 80, 200 - not constant

Test $y \times \sqrt{x}$:

$10 \times \sqrt{4} = 10 \times 2 = 20$

$5 \times \sqrt{16} = 5 \times 4 = 20$

$2 \times \sqrt{100} = 2 \times 10 = 20$

0

b)

$y\sqrt{x} = 20$

$$y = \frac{20}{\sqrt{x}}$$

c)

$$y = \frac{20}{\sqrt{400}} = \frac{20}{20}$$

$y = 1$

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10 y is inversely proportional to the square root of x.

When $x = 2.25$, $y = 4$.

a. Find the value of y when $x = 0.09$.

b. Find the value of x when $y = 15$.

c. Find the value of x for which $y = x$. Give your answer correct to 3 significant figures. (5 marks)

Find k first

$y = \frac{k}{\sqrt{x}}$

$4 = \frac{k}{\sqrt{2.25}} = \frac{k}{1.5}$

$\times 1.5$ $\quad k = 6$ $\quad \times 1.5$

$y = \frac{6}{\sqrt{x}}$

a)

$y = \frac{6}{\sqrt{0.09}} = \frac{6}{0.3}$

$\boxed{y = 20}$

b)

$15 = \frac{6}{\sqrt{x}}$

$\sqrt{x} = \frac{6}{15} = 0.4$

$x = 0.4^2$

$\boxed{x = 0.16}$

c)

$x = \frac{6}{\sqrt{x}}$

$\times \sqrt{x}$ $\quad x\sqrt{x} = 6$ $\quad \times \sqrt{x}$

Square both sides

$x^2 \times x = 36$

$x^3 = 36$

$x = \sqrt[3]{36} = 3.3019...$

$\boxed{x = 3.30}$

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