

Rearranging formulae where the subject appears twice

Worked solutions

GCSE Maths topic practice



This topic online

- 1 Make x the subject of

$$4(x + a) = 3x + b$$

(3 marks)

$$\begin{array}{rclcl} 4(x + a) & = & 3x + b & & \\ 4x + 4a & = & 3x + b & & \\ -3x & & & & -3x \\ x + 4a & = & b & & \\ -4a & & & & -4a \\ x & = & b - 4a & & \end{array}$$

$$\boxed{x = b - 4a}$$

- 2 Assume $a \neq b$.

Make x the subject of

$$ax = bx + c$$

(3 marks)

Collect the x terms on one side

$$ax - bx = c$$

Factorise

$$x(a - b) = c$$

Divide by $(a - b)$

$$x = \frac{c}{a - b}$$

Assume $a \neq b$. Dividing by zero is not allowed

$$\boxed{x = \frac{c}{a - b}}$$

3

Make t the subject of

$$w = \frac{5(t+3)}{t}$$

(3 marks)

$$\times t \quad wt \neq 5(t+3) = 5t + 15 \quad \times t$$

Collect the t terms

$$wt - 5t = 15$$

Factorise

$$t(w - 5) = 15$$

Divide by $(w - 5)$

$$t = \frac{15}{w-5}$$

Here $t \neq 0$ and $w \neq 5$. Dividing by zero is not allowed

$$\boxed{t = \frac{15}{w-5}}$$

4

Make r the subject of

$$p = \frac{2(r+h)}{r}$$

(4 marks)

$$p \neq \frac{2(r+h)}{r}$$

$$\times r \quad pr \neq 2(r+h) \quad \times r$$

$$pr \neq 2r + 2h$$

$$-2r \quad pr - 2r \neq 2h \quad -2r$$

Factorise the left side

$$r(p - 2) \neq 2h$$

$$\div (p-2) \quad r \neq \frac{2h}{(p-2)} \quad \div (p-2)$$

$$\boxed{r = \frac{2h}{(p-2)}}$$

5 Make m the subject of

$$k = \frac{3m - 2}{m + 4}$$

(4 marks)

$$k(m + 4) = 3m - 2$$

Expand

$$km + 4k = 3m - 2$$

Collect the m terms on one side

$$km - 3m = -2 - 4k, \text{ so } 4k + 2 = 3m - km$$

Factorise

$$4k + 2 = m(3 - k)$$

Divide by $(3 - k)$

$$m = \frac{4k + 2}{3 - k}$$

Here $m \neq -4$ and $k \neq 3$. Dividing by zero is not allowed

$$m = \frac{4k + 2}{3 - k}$$

6 Assume $a \neq 5$.

Make p the subject of

$$5(p - q) = ap + 7$$

(4 marks)

Expand the bracket

$$5p - 5q = ap + 7$$

Collect the p terms on one side

$$5p - ap = 7 + 5q$$

Factorise

$$p(5 - a) = 5q + 7$$

Divide by $(5 - a)$

$$p = \frac{5q + 7}{5 - a}$$

Assume $a \neq 5$. Dividing by zero is not allowed

$$p = \frac{5q + 7}{5 - a}$$

7 Make x the subject of

$$y = \frac{2x + 5}{x - 3}$$

(4 marks)

$$\begin{array}{rcl}
 y & = & \frac{2x + 5}{x - 3} \\
 \times(x-3) & & \times(x-3) \\
 y(x-3) & = & 2x + 5 \\
 xy - 3y & = & 2x + 5 \\
 -2x & & -2x \\
 xy - 2x - 3y & = & 5 \\
 +3y & & +3y \\
 xy - 2x & = & 5 + 3y
 \end{array}$$

Factorise the left side

$$\begin{array}{rcl}
 x(y-2) & = & 3y + 5 \\
 \div(y-2) & & \div(y-2) \\
 x & = & \frac{3y + 5}{y - 2}
 \end{array}$$

$$x = \frac{3y + 5}{y - 2}$$

8 Make v the subject of

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

(4 marks)

$$-\frac{1}{u} \quad \frac{1}{v} \quad \Bigg| \quad \frac{1}{f} - \frac{1}{u} \quad -\frac{1}{u}$$

Write the right hand side as a single fraction

$$\frac{1}{v} = \frac{u - f}{fu}$$

Take the reciprocal of both sides

$$v = \frac{fu}{u - f}$$

Check with $f = 2, u = 3: v = \frac{6}{1} = 6$, and $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

Here f and u are nonzero and $u \neq f$. Dividing by zero is not allowed

$$v = \frac{fu}{u - f}$$

9

Make x the subject of

$$y = \sqrt{\frac{x+1}{x-1}}$$

(4 marks)

$$y^2 = \frac{x+1}{x-1}$$

Multiply by $(x-1)$

$$y^2(x-1) = x+1, \text{ so } y^2x - y^2 = x+1$$

Collect the x terms

$$y^2x - x = y^2 + 1$$

Factorise

$$x(y^2 - 1) = y^2 + 1$$

Divide by $(y^2 - 1)$

$$x = \frac{y^2 + 1}{y^2 - 1}$$

Check with $x = 3$: $\frac{3+1}{3-1} = 2$, $y = \sqrt{2}$, and $\frac{2+1}{2-1} = 3$

Because y is a square root, $y \geq 0$. Also $y \neq 1$, so $y^2 - 1$ is nonzero. Dividing by zero is not allowed

$$x = \frac{y^2 + 1}{y^2 - 1}$$

10

Make v the subject of

$$T = \sqrt{\frac{2v}{v-3}}$$

(4 marks)

$$T^2 = \frac{2v}{v-3}$$

Multiply by $(v-3)$

$$T^2 v - 3T^2 = 2v$$

Collect the v terms

$$T^2 v - 2v = 3T^2$$

Factorise

$$v(T^2 - 2) = 3T^2$$

Divide by $(T^2 - 2)$

$$v = \frac{3T^2}{T^2 - 2}$$

Check with $v = 6$: $\frac{2(6)}{6-3} = 4$, $T = 2$, and $\frac{3(4)}{4-2} = 6$

Because T is a square root, $T \geq 0$. Also $T \neq \sqrt{2}$, so $T^2 - 2$ is nonzero. Dividing by zero is not allowed.

$$v = \frac{3T^2}{T^2 - 2}$$

11

Make x the subject of

$$y = \frac{x}{x+2} + 3$$

(5 marks)

$$\begin{array}{rcl}
 y & = & \frac{x}{x+2} + 3 \\
 -3 & & \\
 y - 3 & = & \frac{x}{x+2} \\
 \times(x+2) & & \\
 (y-3)(x+2) & = & x \\
 xy - 3x + 2y - 6 & = & x \\
 -x & & \\
 xy - 4x + 2y - 6 & = & 0 \\
 -2y, +6 & & \\
 xy - 4x & = & 6 - 2y \\
 \text{Factorise the left side} & & \\
 x(y-4) & = & 6 - 2y \\
 \div(y-4) & & \div(y-4) \\
 x & = & \frac{6-2y}{y-4} \\
 \boxed{x = \frac{6-2y}{y-4}}
 \end{array}$$