



- 1 In triangle ABC, angle BAC = 35° , angle ABC = 78° and BC = 9.4 cm.
Work out the length of AC. Give your answer correct to 3 significant figures.

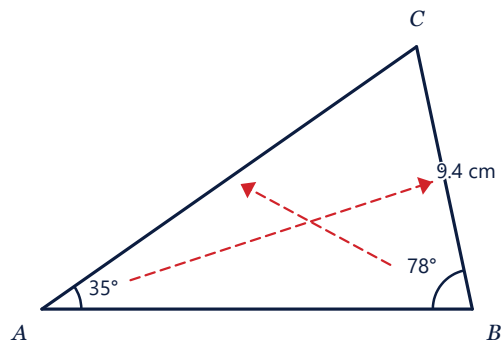


Diagram NOT accurately drawn

(3 marks)

AC is opposite angle B and BC is opposite angle A

Sine rule

$$\frac{AC}{\sin B} = \frac{BC}{\sin A}$$

$$AC = 9.4 \times \sin 78^\circ \div \sin 35^\circ$$

$$AC = 9.4 \times 0.97814... \div 0.57357... = 16.030..$$

$$AC = 16.0 \text{ cm (3 sf)}$$

16.0 cm

2

In triangle ABC, angle BAC = 52° , angle ACB = 63° and AC = 11 cm.
Calculate the length of BC. Give your answer correct to 3 significant figures.

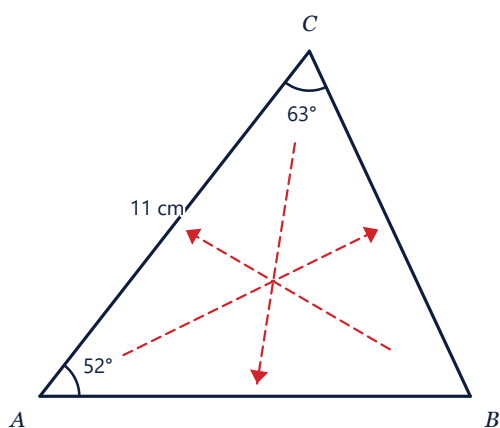


Diagram NOT accurately drawn

(3 marks)

$$\text{Angle } ABC = 180^\circ - 52^\circ - 63^\circ = 65^\circ$$

BC is opposite angle A and AC is opposite angle B

Sine rule

$$\frac{BC}{\sin A} = \frac{AC}{\sin B}$$

$$BC = 11 \times \sin 52^\circ \div \sin 65^\circ = 11 \times 0.78801... \div 0.90630..$$

$$BC = 9.564... = 9.56 \text{ cm (3 s.f.)}$$

$$\boxed{9.56 \text{ cm}}$$

3

In triangle ABC, $AB = 12.5$ cm, $BC = 8.4$ cm and angle $BAC = 33^\circ$.

Angle ACB is acute.

Work out the size of angle ACB. Give your answer correct to 1 decimal place.

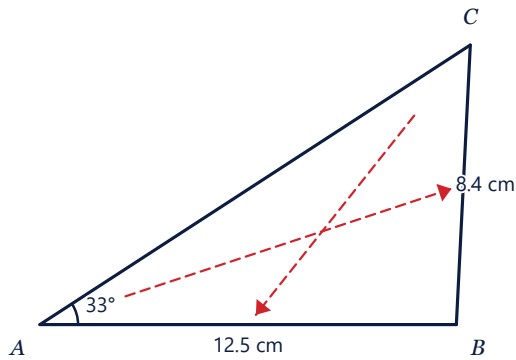


Diagram NOT accurately drawn

(4 marks)

AB is opposite angle C and BC is opposite angle A

Sine rule

$$\frac{\sin C}{AB} = \frac{\sin A}{BC}$$

$$\sin C = 12.5 \times \sin 33^\circ \div 8.4 = 0.81047..$$

$$C = \sin^{-1}(0.81047...) = 54.142...^\circ$$

$$\text{Angle ACB} = 54.1^\circ \text{ (1 dp)}$$

54.1

4

In triangle PQR, QR = 14.2 cm, PR = 9.6 cm and angle QPR = 81° .

Calculate the size of angle PQR. Give your answer correct to 1 decimal place.

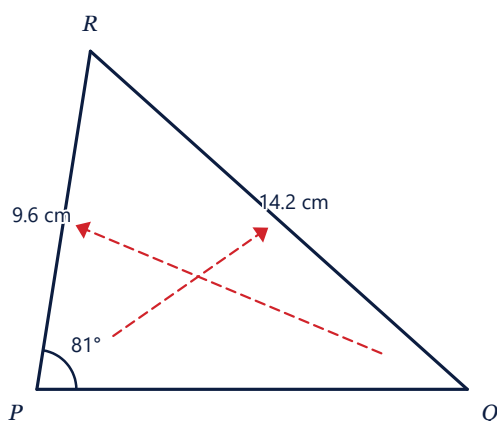


Diagram NOT accurately drawn

(4 marks)

QR is opposite angle P and PR is opposite angle Q

Sine rule

$$\frac{\sin Q}{PR} = \frac{\sin P}{QR}$$

$$\sin Q = 9.6 \times \sin 81^\circ \div 14.2 = 0.66773\dots$$

$$Q = \sin^{-1}(0.66773\dots) = 41.892\dots^\circ$$

Angle PQR = 41.9° (1 dp). The other angle with this sine is $180^\circ - 41.892\dots^\circ = 138.107\dots^\circ$, but $138.107\dots^\circ + 81^\circ$ exceeds 180° , so it cannot fit in this triangle

41.9°

5

B is due east of A. $AB = 250$ m.

The bearing of a mast C from A is 040° .

The bearing of C from B is 300° .

Work out the distance AC. Give your answer correct to 3 significant figures.

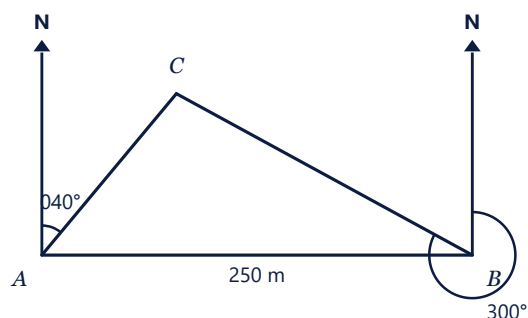


Diagram NOT accurately drawn

(4 marks)

At A

the bearing of C is 040° and B is due east (bearing 090°), so angle $CAB = 90^\circ$

$$\begin{array}{r} - \\ 40^\circ \\ = \\ 50^\circ \end{array}$$

At B

A is due west (bearing 270°) and the bearing of C is 300° , so angle $CBA = 300^\circ$

$$\begin{array}{r} - \\ 270^\circ \\ = \\ 30^\circ \end{array}$$

Angle $ACB = 180^\circ$

$$\begin{array}{r} - \\ 50^\circ \\ - \\ 30^\circ \\ = \\ 100^\circ \end{array}$$

Sine rule

$$\frac{AC}{\sin(CBA)} = \frac{AB}{\sin(ACB)}, \text{ so } AC = 250 \times \sin 30^\circ \div \sin 100^\circ$$

$$AC = 125 \div 0.98480... = 126.928... = 127 \text{ m (3 s.f.)}$$

127 m

6

In triangle ABC, angle ABC = 45° , angle ACB = 60° and $AB = 6\sqrt{2}$ cm.
Without using a calculator, show that $AC = 4\sqrt{3}$ cm.

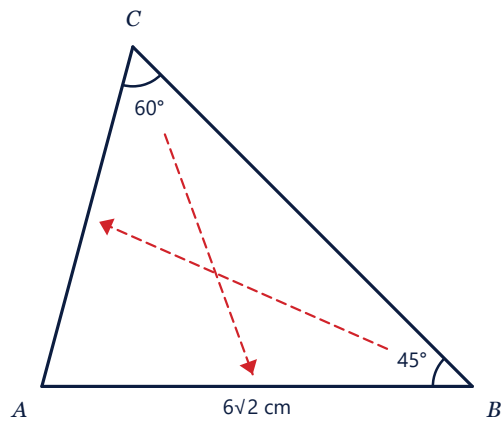


Diagram NOT accurately drawn

(4 marks)

$$\begin{aligned}\frac{AC}{\sin 45^\circ} &= \frac{AB}{\sin 60^\circ} \\ \frac{AC}{\frac{\sqrt{2}}{2}} &= \frac{6\sqrt{2}}{\frac{\sqrt{3}}{2}} \\ AC &= 6\sqrt{2} \times \frac{\sqrt{2}}{2} \div \frac{\sqrt{3}}{2} \\ AC &= 6 \div \frac{\sqrt{3}}{2} \\ AC &= \frac{12}{\sqrt{3}}\end{aligned}$$

Rationalise the denominator

$$AC = \frac{12\sqrt{3}}{3}$$

$$\boxed{AC = 4\sqrt{3} \text{ cm}}$$

7

In triangle ABC, angle BAC = 30° , BC = 5 cm and AB = 8 cm.

Work out the exact value of $\sin C$.

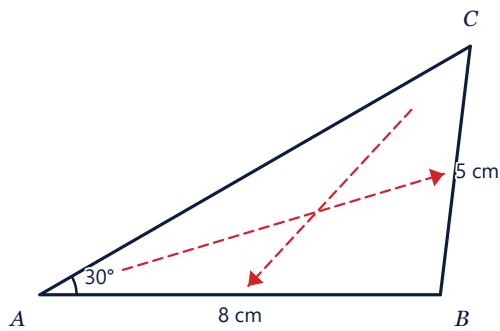


Diagram NOT accurately drawn

(4 marks)

BC is opposite angle A and AB is opposite angle C

Sine rule

$$\frac{\sin C}{AB} = \frac{\sin A}{BC}$$

$$\sin C = 8 \times \sin 30^\circ \div 5$$

$$\sin 30^\circ = \frac{1}{2}, \text{ so } \sin C = 8 \times \frac{1}{2} \div 5 = \frac{4}{5}$$

$$\boxed{\frac{4}{5}}$$

8

In triangle ABC, AB = 10 cm, BC = 7 cm and angle BAC = 40° .

Angle ACB could be acute or obtuse. Work out both possible sizes of angle ACB. Give each answer correct to 1 decimal place.

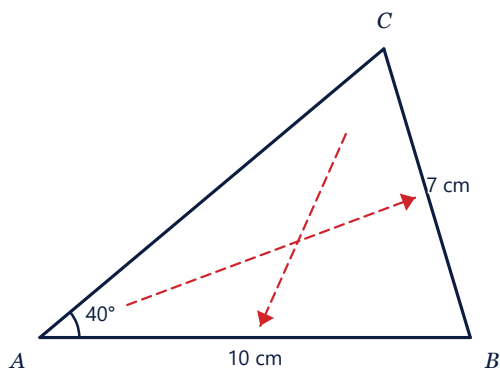


Diagram NOT accurately drawn

(4 marks)

$$\frac{\sin C}{10} = \frac{\sin 40^\circ}{7}$$

$$\sin C = \frac{10 \times \sin 40^\circ}{7}$$

$$\sin C = 0.9183\dots$$

Acute angle

$$C = \sin^{-1}(0.9183)$$

$$C = 66.7^\circ$$

The sine curve is symmetrical about 90° , so there is a second angle

$$C = 180^\circ - 66.7^\circ$$

$$\boxed{66.7^\circ \text{ or } 113.3^\circ}$$

9

A triangular sail has angles of 71° and 44° .

The side opposite the 71° angle has length 6.4 m.

Work out the length of the side opposite the 44° angle. Give your answer correct to 3 significant figures.

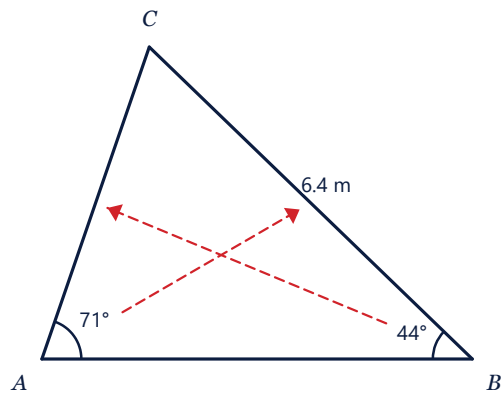


Diagram NOT accurately drawn

(4 marks)

Sine rule

$$\frac{x}{\sin 44^\circ} = \frac{6.4}{\sin 71^\circ}$$

$$x = 6.4 \times \sin 44^\circ \div \sin 71^\circ$$

$$x = 6.4 \times 0.69465... \div 0.94551... = 4.7019..$$

$$x = 4.70 \text{ m (3 sf)}$$

$$\boxed{4.70 \text{ m}}$$

10

A boat leaves harbour H and sails 8 km on a bearing of 070° to a buoy B. A lighthouse L is due north of H. From B, the bearing of L is 320° .

Sketch the triangle HBL, then work out the distance HL. Give your answer correct to 3 significant figures.

(5 marks)

Angles in triangle HBL

Angle at H = 70° (bearing 070° from north)

Bearing of H from B = $070^\circ + 180^\circ = 250^\circ$

Angle at B = $320^\circ - 250^\circ = 70^\circ$

Angle at L = $180^\circ - 70^\circ - 70^\circ = 40^\circ$

Sine rule

$$\frac{HL}{\sin 70^\circ} = \frac{8}{\sin 40^\circ}$$

$$HL = \frac{8 \times \sin 70^\circ}{\sin 40^\circ}$$

$$\boxed{HL = 11.7 \text{ km}}$$

11

In triangle ABC, angle BAC = 25° , BC = 7.2 cm and AB = 13.5 cm.

Angle ACB is obtuse.

Work out the size of angle ACB. Give your answer correct to 1 decimal place.

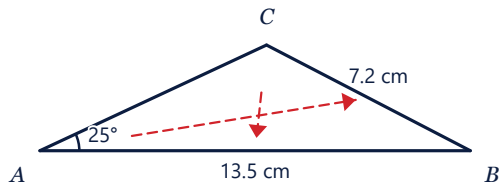


Diagram NOT accurately drawn

(4 marks)

BC is opposite angle A and AB is opposite angle C

Sine rule

$$\sin C = 13.5 \times \sin 25^\circ \div 7.2 = 0.79240..$$

$$\sin^{-1}(0.79240...) = 52.411...^\circ, \text{ but angle ACB is obtuse}$$

Sine is also positive in the second quadrant

$$C = 180^\circ - 52.411...^\circ = 127.588...^\circ$$

Check

$$25^\circ + 127.6^\circ = 152.6^\circ < 180^\circ, \text{ so this is a valid triangle}$$

$$\text{Angle ACB} = 127.6^\circ \text{ (1 dp)}$$

$$\boxed{127.6^\circ}$$

12

The diagram shows quadrilateral ABCD. The diagonal BD splits it into triangles ABD and BCD.

In triangle ABD, angle DAB = 58° , angle ABD = 47° and AD = 9.3 cm.

In triangle BCD, angle DBC = 39° and angle BCD = 64° .

Work out the length of CD. Give your answer correct to 3 significant figures.

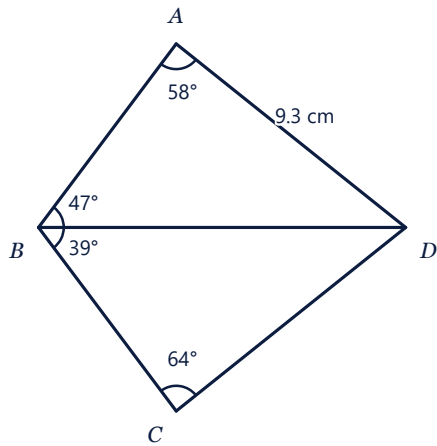


Diagram NOT accurately drawn

(5 marks)

In triangle ABD, side AD is opposite angle ABD = 47° . Side BD is opposite angle DAB = 58° . Always pair a side with its opposite angle

Sine rule

$$BD / \sin 58^\circ = 9.3 / \sin 47^\circ$$

$BD = 9.3 \times \sin 58^\circ \div \sin 47^\circ = 10.7839028869... \text{ cm}$. Keep this full value for the next step

In triangle BCD, CD is opposite 39° and BD is opposite 64° , so $CD / \sin 39^\circ = BD / \sin 64^\circ$

$$CD = 10.7839028869... \times \sin 39^\circ \div \sin 64^\circ = 7.5507064268... \text{ cm}$$

Therefore $CD = 7.55 \text{ cm}$ to 3 significant figures

7.55 cm

13

In triangle ABC, angle BAC = 50° , angle ABC = 70° and AC = 12 cm.
Work out the area of triangle ABC. Give your answer correct to 3 significant figures.

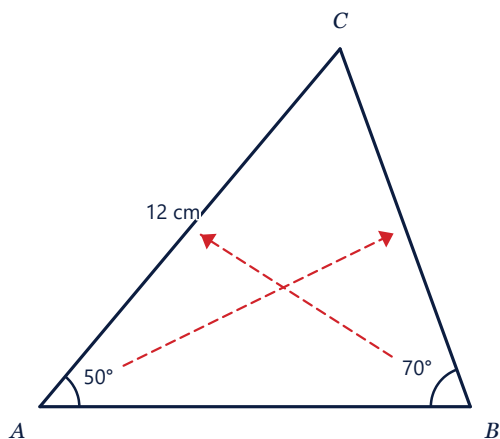


Diagram NOT accurately drawn

(5 marks)

$$\text{angle } C = 180^\circ - 50^\circ - 70^\circ = 60^\circ$$

Sine rule to find BC

$$\frac{BC}{\sin 50^\circ} = \frac{12}{\sin 70^\circ}$$

$$BC = \frac{12 \times \sin 50^\circ}{\sin 70^\circ} = 9.782... \text{ cm}$$

Area = $\frac{1}{2}ab \sin C$

$$\text{Area} = \frac{1}{2} \times 12 \times 9.782... \times \sin 60^\circ$$

$$\boxed{\text{Area} = 50.8 \text{ cm}^2}$$

- 14 In triangle ABC, AB = 12 cm, angle BAC = 40° and angle ABC = 75°.

Work out the perimeter of the triangle. Give your answer correct to 3 significant figures.

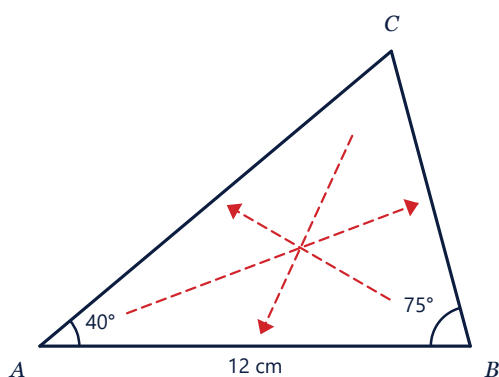


Diagram NOT accurately drawn

(5 marks)

$$\text{Angle } ACB = 180^\circ - 40^\circ - 75^\circ = 65^\circ$$

Sine rule

$$\frac{BC}{\sin A} = \frac{AB}{\sin C}, \text{ so } BC = 12 \times \sin 40^\circ \div \sin 65^\circ = 8.5108..$$

$$\frac{AC}{\sin B} = \frac{AB}{\sin C}, \text{ so } AC = 12 \times \sin 75^\circ \div \sin 65^\circ = 12.789..$$

$$\text{Perimeter} = 12 + 8.5108... + 12.789... = 33.300..$$

$$\text{Perimeter} = 33.3 \text{ cm (3 s.f.)}$$

$$\boxed{33.3 \text{ cm}}$$

- 15 In triangle ABC, the point D lies on the side BC. The line AD bisects angle BAC.

Use the sine rule in triangles ABD and ACD to prove that $\frac{BD}{DC} = \frac{AB}{AC}$

(4 marks)

Let angle BAD = angle DAC = θ and angle ADB = ϕ

Angle ADC = $180^\circ - \phi$ (angles on a straight line)

Sine rule in triangle ABD

$$\frac{BD}{\sin \theta} = \frac{AB}{\sin \phi}$$

$$BD = \frac{AB \sin \theta}{\sin \phi}$$

Sine rule in triangle ACD

$$\frac{DC}{\sin \theta} = \frac{AC}{\sin(180^\circ - \phi)}$$

$$\sin(180^\circ - \phi) = \sin \phi$$

$$DC = \frac{AC \sin \theta}{\sin \phi}$$

Divide BD by DC: $\sin \theta$ and $\sin \phi$ cancel

$$\boxed{\frac{BD}{DC} = \frac{AB}{AC}}$$