



- 1** Write $\frac{x+1}{2} + \frac{2x-1}{5}$ as a single fraction in its simplest form.

(2 marks)

$$\begin{aligned}\frac{x+1}{2} + \frac{2x-1}{5} &= \frac{5(x+1)}{10} + \frac{2(2x-1)}{10} \\ &= \frac{5x+5+4x-2}{10}\end{aligned}$$

$$\boxed{\frac{9x+3}{10}}$$

- 2** Write $\frac{3}{x+1} - \frac{2}{x+4}$ as a single fraction. Simplify the numerator.

(3 marks)

$$\begin{aligned}\frac{3}{x+1} - \frac{2}{x+4} &= \frac{3(x+4) - 2(x+1)}{(x+1)(x+4)} \\ &= \frac{3x+12-2x-2}{(x+1)(x+4)}\end{aligned}$$

$$\boxed{\frac{x+10}{(x+1)(x+4)}}$$

3

Show that

$$\frac{4}{x+3} - \frac{2}{x+5}$$

can be written as $\frac{2(x+7)}{(x+3)(x+5)}$

(3 marks)

The original expression is defined only when $x \neq -3$ and $x \neq -5$. A denominator cannot be zero. These restrictions still apply after simplifying

Make the denominators match by multiplying each fraction's numerator and denominator by the same missing factor. This multiplies each fraction by 1, so its value stays the same

Use the common denominator $(x+3)(x+5)$

$$\frac{4}{x+3} - \frac{2}{x+5} = \frac{4(x+5) - 2(x+3)}{(x+3)(x+5)}$$

Expand the numerator

$$4x + 20 - 2x - 6 = 2x + 14$$

Factorise

$$2x + 14 = 2(x+7), \text{ giving } \frac{2(x+7)}{(x+3)(x+5)} \text{ as required}$$

$$\boxed{\frac{2(x+7)}{(x+3)(x+5)}}$$

4

Solve

$$\frac{x+4}{3} + \frac{2x-1}{5} = 7$$

(3 marks)

Multiply every term by 15

$$5(x+4) + 3(2x-1) = 105$$

Expand

$$5x + 20 + 6x - 3 = 105$$

Simplify

$$11x + 17 = 105, \text{ so } 11x = 88$$

$$x = 8. \text{ Check: } \frac{12}{3} + \frac{15}{5} = 4 + 3 = 7$$

$$\boxed{x = 8}$$

5

Solve

$$\frac{4}{x-1} - \frac{2}{x+2} = 1$$

(4 marks)

The original expression is defined only when $x \neq 1$ and $x \neq -2$. A denominator cannot be zero. These restrictions still apply after simplifying

$$x(x-1)(x+2) \quad 4(x+2) - 2(x-1) \neq (x-1)(x+2) \quad x(x-1)(x+2)$$

Expand

$$4x + 8 - 2x + 2 = x^2 + x - 2, \text{ so } 2x + 10 = x^2 + x - 2$$

Rearrange

$$x^2 - x - 12 = 0$$

Factorise

$$(x-4)(x+3) = 0, \text{ so } x = 4 \text{ or } x = -3$$

$$\text{Check } x = 4: \frac{4}{3} - \frac{2}{6} = \frac{4}{3} - \frac{1}{3} = 1. \text{ Check } x = -3: \frac{4}{-4} - \frac{2}{-1} = -1 + 2 = 1$$

$$2 = 1$$

$$x = 4 \text{ or } x = -3$$

6

Solve

$$\frac{1}{x-2} + \frac{2}{x+1} = 3$$

Give your answers in the form $a \pm \sqrt{b}$ where a and b are integers.

(5 marks)

The original expression is defined only when $x \neq 2$ and $x \neq -1$. A denominator cannot be zero. These restrictions still apply after simplifying

$$x(x-2)(x+1) \quad (x+1) + 2(x-2) \neq 3(x-2)(x+1) \quad x(x-2)(x+1)$$

Expand

$$3x - 3 = 3(x^2 - x - 2)$$

Divide by 3

$$x - 1 = x^2 - x - 2, \text{ so } x^2 - 2x - 1 = 0$$

Complete the square

$$(x-1)^2 = 2$$

$$x = 1 \pm \sqrt{2}. \text{ Check } x = 1 + \sqrt{2}: \frac{1}{\sqrt{2}-1} + \frac{2}{2+\sqrt{2}} = (\sqrt{2}+1) + (2-\sqrt{2}) = 3$$

$$(2 - \sqrt{2}) = 3$$

$$x = 1 + \sqrt{2} \text{ or } x = 1 - \sqrt{2}$$

7

Show that

$$\frac{x^2 + 3x}{x^2 - x - 12} - \frac{2}{x + 1}$$

can be written as $\frac{x^2 - x + 8}{(x - 4)(x + 1)}$

(6 marks)

The original expression is defined only when $x \neq -3, 4$ or -1 . A denominator cannot be zero. These restrictions still apply after simplifying

Factorise the first fraction

$$x^2 + 3x = x(x + 3) \text{ and } x^2 - x - 12 = (x - 4)(x + 3)$$

Cancel $(x + 3)$: the first fraction simplifies to $\frac{x}{x - 4}$

Combine over the common denominator $(x - 4)(x + 1)$:

$$\frac{x(x + 1) - 2(x - 4)}{(x - 4)(x + 1)}$$

Expand the numerator

$$x^2 + x - 2x + 8 = x^2 - x + 8$$

This gives $\frac{x^2 - x + 8}{(x - 4)(x + 1)}$ as required

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

$$\boxed{\frac{x^2 - x + 8}{(x - 4)(x + 1)}}$$

8

a. Simplify fully $\frac{2x^2 + 5x - 3}{x^2 - 9}$

b. Hence, or otherwise, solve

$$\frac{2x^2 + 5x - 3}{x^2 - 9} = 5$$

(6 marks)

The original expression is defined only when $x \neq 3$ and $x \neq -3$. A denominator cannot be zero. These restrictions still apply after simplifying

a) Factorise

$$2x^2 + 5x - 3 = (2x - 1)(x + 3) \text{ and } x^2 - 9 = (x - 3)(x + 3)$$

a) Cancel $(x + 3)$ to get $\frac{2x - 1}{x - 3}$

$$\text{b) Using part a, solve } \frac{2x - 1}{x - 3} = 5$$

b) Multiply up

$$2x - 1 = 5(x - 3) = 5x - 15$$

$$\text{b) So } 14 = 3x, \text{ giving } x = \frac{14}{3}$$

b) Check

$$\text{at } x = \frac{14}{3} \text{ the original fraction is } \frac{575}{9} \frac{115}{9} = 5$$

Cancel common factors only: dividing the whole numerator and denominator by the same nonzero factor leaves the fraction unchanged. You cannot cancel individual terms joined by + or -

9

Write $\frac{2}{x^2 - 9} + \frac{1}{x + 3}$ as a single fraction in its simplest form.

(3 marks)

Factorise the denominator

$$x^2 - 9 = (x - 3)(x + 3)$$

$$= \frac{2}{(x-3)(x+3)} + \frac{x-3}{(x-3)(x+3)}$$

$$= \frac{2 + x - 3}{(x-3)(x+3)}$$

$$\boxed{\frac{x-1}{(x-3)(x+3)}}$$

10

$$\frac{x + 2y}{3x - y} = 2$$

Find the ratio $x : y$. Give your answer in the form $a : b$ where a and b are integers.

(3 marks)

$$\begin{array}{rcl}
 x(3x - y) & x + 2y & = 2(3x - y) \quad x(3x - y) \\
 & x + 2y & = 6x - 2y \\
 +2y - x & 4y & = 5x \quad +2y - x \\
 & \frac{x}{y} & = \frac{4}{5}
 \end{array}$$

$$x : y = 4 : 5$$

11

$$\text{Solve } \frac{3}{x+2} + \frac{4}{x-1} = 1$$

Show all your working.

(5 marks)

$$\begin{array}{rcl}
 x(x+2)(x-1) & 3(x-1) + 4(x+2) & = (x+2)(x-1) \quad x(x+2)(x-1) \\
 & 3x - 3 + 4x + 8 & = x^2 + x - 2 \\
 & 7x + 5 & = x^2 + x - 2 \\
 & 0 & = x^2 - 6x - 7 \\
 & 0 & = (x-7)(x+1)
 \end{array}$$

$$x = 7 \text{ or } x = -1$$