Differentiation

Worked solutions

GCSE Maths topic practice

This topic online

1 $y = x^3 + 4x^2 - 7x + 2$

Find $\frac{dy}{dx}$

(2 marks)

Multiply by the power, take 1 off the power; the +2 disappears

$$\frac{dy}{dx} = 3x^2 + 8x - 7$$

$$\boxed{\frac{dy}{dx} = 3x^2 + 8x - 7}$$

2 $f(x) = 5x^4 - 2x^2 + 9$

a. Find $f'(x)$

b. Work out the value of $f'(-1)$

(3 marks)

a) $f'(x)$ is the derivative; the 9 disappears

$$f'(x) = 20x^3 - 4x$$

$$\boxed{f'(x) = 20x^3 - 4x}$$

b) Substitute $x = -1$

$$f'(-1) = 20 \times (-1)^3 - 4 \times (-1)$$

$$= -20 + 4$$

$$\boxed{f'(-1) = -16}$$

3 The point P (4, −7) lies on the curve with equation $y = x^2 - 6x + 1$

Find the gradient of the curve at P.

(2 marks)

$$\frac{dy}{dx} = 2x - 6$$

Substitute $x = 4$ into $\frac{dy}{dx}$

$$\text{gradient} = 2 \times 4 - 6$$

$$\boxed{\text{gradient} = 2}$$

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4 $y = \frac{6}{x} + x^2$

a. Find $\frac{dy}{dx}$

b. Find the gradient of the curve at the point where $x = 2$ (3 marks)

a) Rewrite the fraction as a negative power first

$$y = 6x^{-1} + x^2$$

$$\frac{dy}{dx} = -6x^{-2} + 2x$$

$$\boxed{\frac{dy}{dx} = 2x - \frac{6}{x^2}}$$

b) Substitute $x = 2$

$$\text{gradient} = 2 \times 2 - \frac{6}{2^2}$$

$$= 4 - 1.5$$

$$\boxed{\text{gradient} = 2.5}$$

5 $y = \frac{4}{x^2} + x$

a. Find $\frac{dy}{dx}$

b. Find the gradient of the curve at the point where $x = -1$ (3 marks)

a) Rewrite as a negative power first

$$y = 4x^{-2} + x$$

$$\frac{dy}{dx} = -8x^{-3} + 1$$

$$\boxed{\frac{dy}{dx} = 1 - \frac{8}{x^3}}$$

b) Substitute $x = -1$, and $(-1)^3 = -1$

$$\text{gradient} = 1 - \frac{8}{(-1)^3}$$

$$= 1 + 8$$

$$\boxed{\text{gradient} = 9}$$

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6 A curve has equation $y = x^3 - 3x^2 + 5x$

Find the x-coordinates of the points on the curve where the gradient is 5. (3 marks)

$$\frac{dy}{dx} = 3x^2 - 6x + 5$$

Gradient is 5, so put $\frac{dy}{dx} = 5$

$$3x^2 - 6x + 5 = 5$$

-5 -5

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

7 The curve with equation $y = x^2 - 8x + 3$ has exactly one point where the gradient is zero.

Find the coordinates of this point. (3 marks)

$$\frac{dy}{dx} = 2x - 8$$

Gradient is zero, so put $\frac{dy}{dx} = 0$

$$2x - 8 = 0$$

$+8$ $+8$

$$2x = 8$$

$\div 2$ $\div 2$

$$x = 4$$

Substitute $x = 4$ into y to find the y-coordinate

$$y = 4^2 - 8 \times 4 + 3 = -13$$

$$(4, -13)$$

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8 Water flows into a tank. The volume of water in the tank, V litres, t minutes after the water starts to flow is given by

$V = 20t + 6t^2 - t^3$ for $0 \leq t \leq 5$

a. Find $\frac{dV}{dt}$

b. Work out the value of $\frac{dV}{dt}$ when $t = 3$

c. Explain, in context, what your answer to part (b) tells you.

(4 marks)

a) Differentiate V with respect to t

$$\frac{dV}{dt} = 20 + 12t - 3t^2$$

$$\boxed{\frac{dV}{dt} = 20 + 12t - 3t^2}$$

b) Substitute $t = 3$

$$\frac{dV}{dt} = 20 + 12 \times 3 - 3 \times 3^2$$

$$= 20 + 36 - 27$$

$$\boxed{29}$$

c) $\frac{dV}{dt}$ is the rate the volume is changing

$$\boxed{\text{At } t = 3 \text{ the water is flowing in at 29 litres per minute}}$$

9 The curve C has equation $y = x^3 - 2x + 1$

Find the equation of the tangent to C at the point where $x = 2$

Give your answer in the form $y = mx + c$

(4 marks)

Find the point: substitute $x = 2$ into y

$$y = 2^3 - 2 \times 2 + 1 = 5$$

Find the gradient from $\frac{dy}{dx}$

$$\frac{dy}{dx} = 3x^2 - 2$$

$$m = 3 \times 2^2 - 2 = 10$$

Line through $(2, 5)$ with gradient 10

$$y - 5 = 10(x - 2)$$

$$y - 5 = 10x - 20$$

$$y = 10x - 15$$

(+5 on both sides)

$$\boxed{y = 10x - 15}$$

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10 The curve C has equation $y = x(x - 3)^2$

a. Find $\frac{dy}{dx}$

b. Find the equation of the tangent to C at the point where $x = 4$

Give your answer in the form $y = mx + c$

(5 marks)

a) Expand the brackets before differentiating

$(x - 3)^2 = x^2 - 6x + 9$

$y = x(x^2 - 6x + 9) = x^3 - 6x^2 + 9x$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$$\boxed{\frac{dy}{dx} = 3x^2 - 12x + 9}$$

b) Find the point: substitute $x = 4$ into y

$y = 4 \times (4 - 3)^2 = 4$

Gradient: substitute $x = 4$ into $\frac{dy}{dx}$

$m = 3 \times 4^2 - 12 \times 4 + 9 = 9$

Line through $(4, 4)$ with gradient 9

$y - 4 = 9(x - 4)$

$y - 4 = 9x - 36$

$+4 \quad y = 9x - 32 \quad +4$

$$\boxed{y = 9x - 32}$$

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