

Equation of a circle centred at the origin

Worked solutions

GCSE Maths topic practice



This topic online

- 1 A circle has its centre at the origin O and has radius 7.

Write down the equation of the circle.

(1 mark)

$$\text{Centre O: } x^2 + y^2 = r^2$$

$$r^2 = 7^2 = 49$$

$$x^2 + y^2 = 49$$

- 2 A circle has equation $x^2 + y^2 = 81$

Write down the radius of the circle.

(1 mark)

The circle $x^2 + y^2 = r^2$ has centre the origin and radius r

Here $r^2 = 81$, so $r = 9$

$$9$$

- 3 A circle has its centre at the origin O. The diameter of the circle is 16.

Find the equation of the circle.

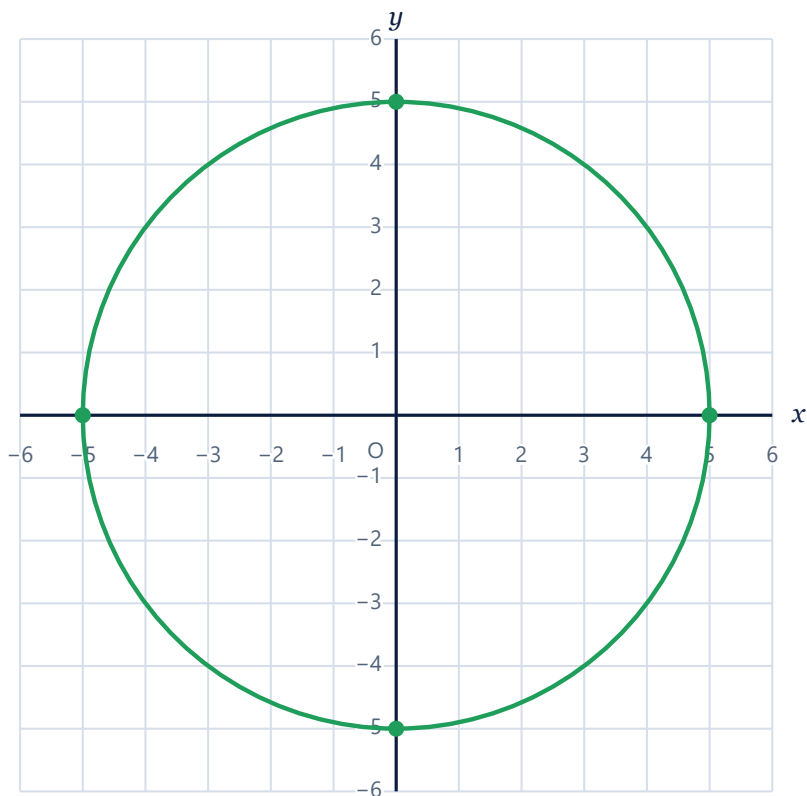
(2 marks)

$$\text{radius} = 16 \div 2 = 8$$

$$r^2 = 8^2 = 64$$

$$x^2 + y^2 = 64$$

- 4 On the grid, draw the circle with equation $x^2 + y^2 = 25$.



(2 marks)

$$x^2 + y^2 = r^2, \text{ so } r^2 = 25$$

$$r = 5$$

Centre (0, 0). Mark 5 units out on each axis

Circle, centre O, radius 5

5

A circle has equation $x^2 + y^2 = 45$.

For each point, state whether it lies inside the circle, on the circle or outside the circle. Show your working.

- a. P (6, 3)
- b. Q (5, -4)
- c. R (-7, 1)

(3 marks)

Work out $x^2 + y^2$ and compare with 45

a)

$$6^2 + 3^2 = 45$$

= 45, so on the circle

b)

$$5^2 + (-4)^2 = 41$$

< 45, so inside

c)

$$(-7)^2 + 1^2 = 50$$

> 45, so outside

a) on b) inside c) outside

6

The circle C has equation $x^2 + y^2 = 61$

- a. Show that the point A (5, -6) lies on C.
- b. Does the point B (4, 7) lie on C? You must justify your answer.

(3 marks)

a) Substitute A into the left-hand side

$$5^2 + (-6)^2 = 25 + 36 = 61. \text{ This equals 61, so A lies on C}$$

b) Substitute B

$$4^2 + 7^2 = 16 + 49 = 65. \text{ Since 65 is not equal to 61, B does not lie on C. In fact } 65 > 61, \text{ so B is outside the circle}$$

7 A circle has equation $x^2 + y^2 = 100$.

a. A point on the circle has x -coordinate 6 and a positive y -coordinate. Work out its y -coordinate.

b. A point on the circle has y -coordinate -8 . Work out both possible x -coordinates.

(3 marks)

a)

$$\begin{array}{rcl} 6^2 + y^2 & = & 100 \\ -36 & & -36 \\ y^2 & = & 64 \end{array}$$

$$y = 8$$

b)

$$\begin{array}{rcl} x^2 + 64 & = & 100 \\ -64 & & -64 \\ x^2 & = & 36 \end{array}$$

$$x = 6 \text{ or } x = -6$$

a) 8 b) 6 and -6

8 The point $(k, 2k)$ lies on the circle with equation $x^2 + y^2 = 45$.

Work out the two possible values of k .

(3 marks)

$$\begin{array}{rcl} k^2 + (2k)^2 & = & 45 \\ 5k^2 & = & 45 \\ \div 5 & & \div 5 \\ k^2 & = & 9 \end{array}$$

$$k = 3 \text{ or } k = -3$$

9 A circle has centre O, the origin, and passes through the point T (5, 12).

a. Find an equation of the circle.

b. The point D has coordinates (10, 9). Decide whether D is inside the circle, on the circle, or outside the circle. You must show your working.

(4 marks)

a) The radius squared is $OT^2 = 5^2 + 12^2 = 25 + 144 = 169$ (radius 13)

So the circle has equation $x^2 + y^2 = 169$

b) For D

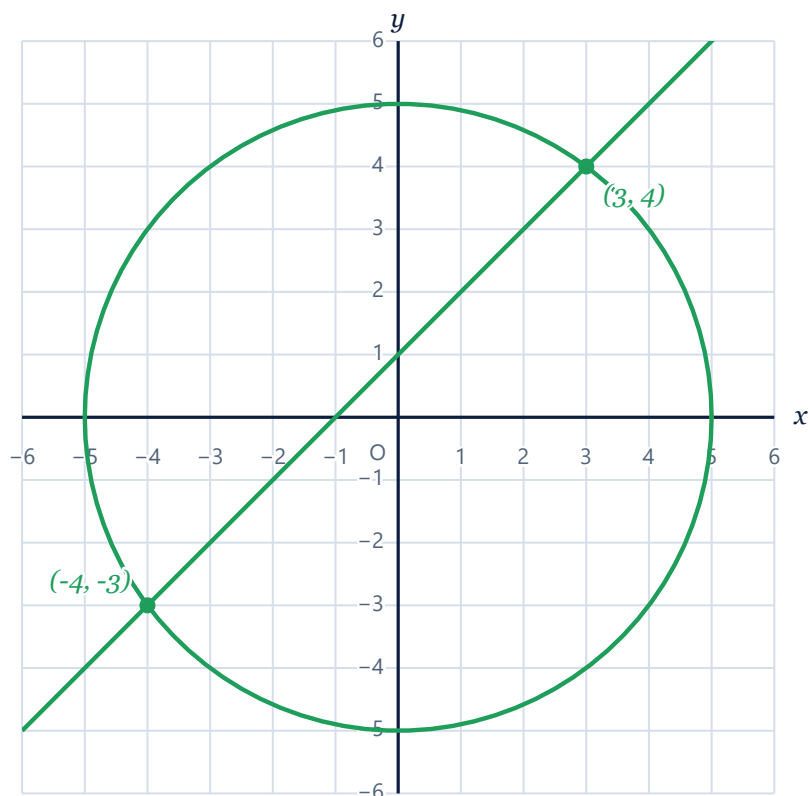
$$10^2 + 9^2 = 100 + 81 = 181$$

$181 > 169$, so D is further from O than the radius: D is outside the circle

- 10 On the grid, draw the circle $x^2 + y^2 = 25$ and the straight line $y = x + 1$.

Use your graphs to solve the simultaneous equations

$$x^2 + y^2 = 25 \quad \text{and} \quad y = x + 1$$



(4 marks)

Circle: centre O , radius 5

Line: through $(0, 1)$, $(2, 3)$, $(-3, -2)$

Read off where they cross

$$\text{check: } 3^2 + 4^2 = 25 \checkmark$$

$$\text{check: } (-4)^2 + (-3)^2 = 25 \checkmark$$

$$\boxed{x = 3, y = 4 \text{ and } x = -4, y = -3}$$

11

The point A (3, 4) lies on the circle with equation $x^2 + y^2 = 25$. The centre of the circle is O (0, 0).

Find the equation of the tangent to the circle at A. Give your answer in the form $ax + by = c$ where a , b and c are integers.

(4 marks)

$$\text{gradient of } OA = \frac{4}{3}$$

Tangent $\perp$ radius: negative reciprocal

$$\text{gradient of tangent} = -\frac{3}{4}$$

$$y - 4 = -\frac{3}{4}(x - 3)$$

 $\times 4$

$$4y - 16 = -3(x - 3) \quad \times 4$$

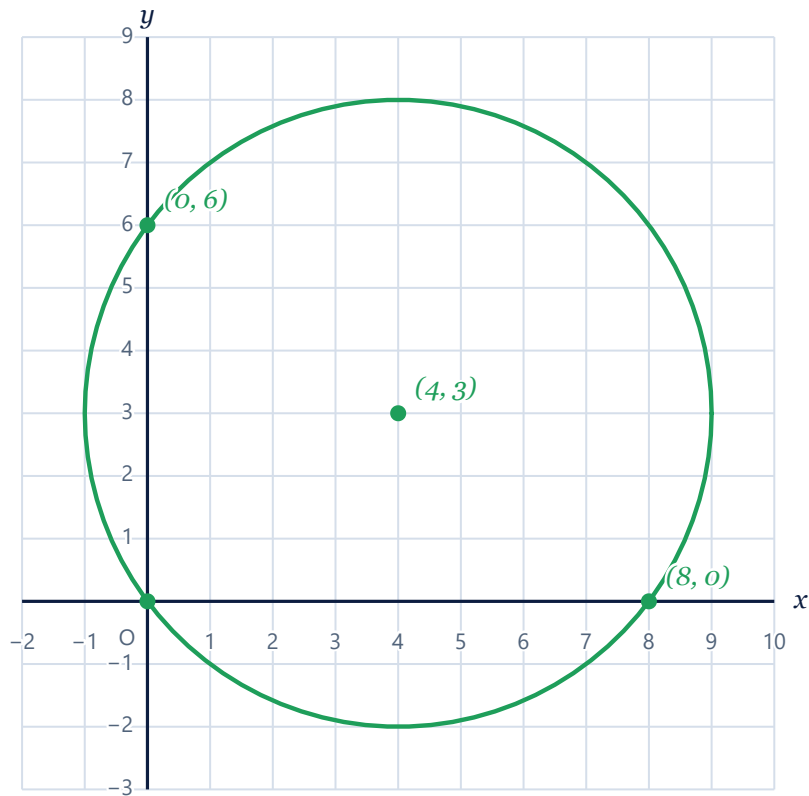
$$4y - 16 = -3x + 9$$

$$\boxed{3x + 4y = 25}$$

12

A circle has equation $(x - 4)^2 + (y - 3)^2 = 25$. Its centre is $(4, 3)$ and its radius is 5.

- On the grid, draw the circle.
- Work out the coordinates of the points where the circle crosses the x -axis.



(4 marks)

a)

Centre $(4, 3)$, radius 5

Points $(9, 3)$, $(-1, 3)$, $(4, 8)$, $(4, -2)$

b)

On the x -axis, $y = 0$

$$\begin{array}{rcl}
 (x - 4)^2 + 9 & = & 25 \\
 -9 & & -9 \\
 (x - 4)^2 & = & 16 \\
 x - 4 & = & \pm 4
 \end{array}$$

$(0, 0)$ and $(8, 0)$

13

The line with equation $y = x + 2$ intersects the circle with equation $x^2 + y^2 = 34$ at the points A and B.

Find the coordinates of A and B.

(5 marks)

$$\text{Substitute } y = x + 2 \text{ into the circle equation: } x^2 + (x + 2)^2 = 34$$

Expand

$$x^2 + x^2 + 4x + 4 = 34, \text{ so } 2x^2 + 4x - 30 = 0$$

Divide by 2

$$x^2 + 2x - 15 = 0, \text{ so } (x + 5)(x - 3) = 0$$

$$x = -5 \text{ gives } y = -3; x = 3 \text{ gives } y = 5$$

Check both points

$$(-5)^2 + (-3)^2 = 25 + 9 = 34 \text{ and } 3^2 + 5^2 = 9 + 25 = 34$$

The points are $(-5, -3)$ and $(3, 5)$

$$\boxed{(-5, -3) \text{ and } (3, 5)}$$