



- 1 Expand and simplify $(6 + \sqrt{11})(6 - \sqrt{11})$

(2 marks)

$$\begin{aligned} &= (6 + \sqrt{11})(6 - \sqrt{11}) \\ &= 36 - 6\sqrt{11} + 6\sqrt{11} - 11 \end{aligned}$$

the surd terms cancel

$$= 36 - 11$$

25

- 2 Expand and simplify $(5 + 2\sqrt{3})(5 - 2\sqrt{3})$

(2 marks)

difference of two squares

$$\begin{aligned} (5 + 2\sqrt{3})(5 - 2\sqrt{3}) &= 5^2 - (2\sqrt{3})^2 \\ &= 25 - 4 \times 3 \\ &= 25 - 12 \end{aligned}$$

13

- 3 Expand and simplify $(2 + \sqrt{5})(3 + \sqrt{5})$

(3 marks)

$$\begin{aligned} \text{Expand the brackets: } &2 \times 3 + 2\sqrt{5} + 3\sqrt{5} + \sqrt{5} \times \sqrt{5} \\ \sqrt{5} \times \sqrt{5} &= 5, \text{ so this is } 6 + 2\sqrt{5} + 3\sqrt{5} + 5 \end{aligned}$$

$$\text{Collect terms: } 11 + 5\sqrt{5}$$

11 + 5√5

- 4 Expand and simplify $(2 + 3\sqrt{2})(4 - \sqrt{2})$

Give your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(3 marks)

$$\begin{aligned} &= (2 + 3\sqrt{2})(4 - \sqrt{2}) \\ &= 8 - 2\sqrt{2} + 12\sqrt{2} - 3\sqrt{2} \times \sqrt{2} \\ &= 8 - 2\sqrt{2} + 12\sqrt{2} - 6 \end{aligned}$$

collect whole numbers and surds

$$= (8 - 6) + (12\sqrt{2} - 2\sqrt{2})$$

2 + 10√2

5 Expand and simplify $(3 - 2\sqrt{5})^2$

Give your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(3 marks)

$$\begin{aligned}(3 - 2\sqrt{5})^2 &= (3 - 2\sqrt{5})(3 - 2\sqrt{5}) \\ &= 9 - 6\sqrt{5} - 6\sqrt{5} + 4 \times 5 \\ &= 9 - 12\sqrt{5} + 20\end{aligned}$$

$$\boxed{29 - 12\sqrt{5}}$$

6 Expand and simplify $(4 - \sqrt{3})^2$

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(3 marks)

$$\begin{aligned}(4 - \sqrt{3})^2 &= 16 - 4\sqrt{3} - 4\sqrt{3} + (\sqrt{3})^2 \\ (\sqrt{3})^2 &= 3, \text{ so this is } 16 - 8\sqrt{3} + 3\end{aligned}$$

Collect terms

$$19 - 8\sqrt{3}, \text{ so } a = 19 \text{ and } b = -8$$

$$\boxed{19 - 8\sqrt{3}}$$

7 Show that $(2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 = 14$

(4 marks)

$$\begin{aligned}(2 + \sqrt{3})^2 &= 4 + 2\sqrt{3} + 2\sqrt{3} + 3 \\ &= 7 + 4\sqrt{3} \\ (2 - \sqrt{3})^2 &= 4 - 2\sqrt{3} - 2\sqrt{3} + 3 \\ &= 7 - 4\sqrt{3}\end{aligned}$$

add the two results

$$(7 + 4\sqrt{3}) + (7 - 4\sqrt{3}) = 14$$

$$\boxed{= 14, \text{ as required}}$$

8 Show that $(\sqrt{6} + \sqrt{2})^2 = 8 + 4\sqrt{3}$

(4 marks)

$$(\sqrt{6} + \sqrt{2})^2 = 6 + 2\sqrt{12} + 2$$

simplify $\sqrt{12}$

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

$$2\sqrt{12} = 4\sqrt{3}$$

$$= 6 + 4\sqrt{3} + 2$$

$$\boxed{8 + 4\sqrt{3}, \text{ as required}}$$

9

A right-angled triangle has perpendicular sides of length $(3 + \sqrt{2})$ cm and $(3 - \sqrt{2})$ cm.

a. Show that the area of the triangle is 3.5 cm^2 .

b. Find the exact length of the hypotenuse.

(5 marks)

a)

$$\begin{aligned}\text{area} &= \frac{1}{2} \times (3 + \sqrt{2})(3 - \sqrt{2}) \\ &= \frac{1}{2} \times (9 - 2) \\ &= \frac{1}{2} \times 7 \\ &= 3.5 \text{ cm}^2\end{aligned}$$

b) Pythagoras

$$\begin{aligned}h^2 &= (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 \\ &= (9 + 6\sqrt{2} + 2) + (9 - 6\sqrt{2} + 2) \\ &= 22\end{aligned}$$

$$h = \sqrt{22} \text{ cm}$$

10

A trapezium has parallel sides of length $(3 + \sqrt{2})$ cm and $(5 + 3\sqrt{2})$ cm.

The perpendicular distance between the parallel sides is $(4 - \sqrt{2})$ cm.

Work out the exact area of the trapezium.

Give your answer in the form $(a + b\sqrt{2})$ cm², where a and b are integers.

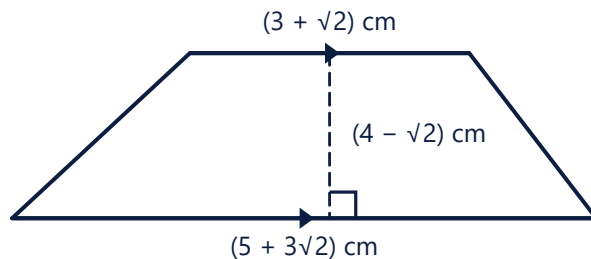


Diagram NOT accurately drawn

(6 marks)

$$\text{Area of a trapezium} = \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$$

Sum of the parallel sides

$$(3 + \sqrt{2}) + (5 + 3\sqrt{2}) = 8 + 4\sqrt{2}$$

Half of this is $4 + 2\sqrt{2}$

$$\text{Area} = (4 + 2\sqrt{2})(4 - \sqrt{2}) = 16 - 4\sqrt{2} + 8\sqrt{2} - 2 \times 2$$

$$= 16 + 4\sqrt{2} - 4 = 12 + 4\sqrt{2}, \text{ so the area is } (12 + 4\sqrt{2})$$

cm²

$12 + 4\sqrt{2}$