

Gradient of a curve and area under a curve

Worked solutions

GCSE Maths topic practice

1

A train leaves a station. Its speed, v m/s, is measured at different times, t seconds.

t	0	2	6	10
v	0	5	9	10

- Use three trapezia to estimate the distance the train travels in the first 10 seconds.
- Work out an estimate of the mean speed over the 10 seconds.

(4 marks)

a)

$$\text{area of trapezium} = \frac{1}{2} \times \text{width} \times (\text{sum of parallel sides})$$

$$0-2: \frac{1}{2} \times 2 \times (0 + 5) = 5$$

$$2-6: \frac{1}{2} \times 4 \times (5 + 9) = 28$$

$$6-10: \frac{1}{2} \times 4 \times (9 + 10) = 38$$

$$\text{total} = 71 \text{ m}$$

b)

$$\text{mean speed} = \{71\} / \{10\}$$

$$\boxed{\text{a) } 71 \text{ m b) } 7.1 \text{ m/s}}$$

2

The temperature, T °C, of a cup of tea is plotted against time, t minutes, and the points form a smooth curve going down.

At $t = 10$ a tangent is drawn to the curve. The tangent passes through the points (2, 80) and (14, 56).

- Work out the gradient of the tangent.
- What does this gradient tell you about the tea at $t = 10$?

(3 marks)

a)

$$\begin{aligned}\text{gradient} &= \{56 - 80\} / \{14 - 2\} \\ &= \{-24\} / \{12\} \\ &= -2\end{aligned}$$

b)

gradient = rate of change of T
negative: temperature is falling

a) -2 b) cooling at 2 °C per minute

3

A runner's velocity-time graph is a smooth curve. The table gives her velocity v m/s at time t seconds.

t : 0, 10, 20, 30 v : 0, 4.2, 6.6, 7.8

Use the trapezium rule with 3 strips of equal width to estimate the distance the runner travels in the first 30 seconds.

(3 marks)

Distance is the area under the velocity-time graph. Use 3 trapezia of width 10

First strip

$$\frac{1}{2} \times 10 \times (0 + 4.2) = 21$$

Second strip

$$\frac{1}{2} \times 10 \times (4.2 + 6.6) = 54$$

Third strip

$$\frac{1}{2} \times 10 \times (6.6 + 7.8) = 72$$

$$\text{Estimate} = 21 + 54 + 72 = 147 \text{ m}$$

147 m

4

A cyclist accelerates away from traffic lights. Her speed-time graph is a smooth curve that gets less steep as time increases. The table gives her speed v m/s at time t seconds.

t : 0, 5, 10, 15, 20 v : 0, 3.6, 5.4, 6.4, 7.0

- Use the trapezium rule with 4 strips of equal width to estimate the distance travelled in the first 20 seconds.
- State, with a reason, whether your answer to part (a) is an underestimate or an overestimate of the actual distance.

(4 marks)

$$\begin{aligned} \text{a) Area} &= \frac{1}{2} \times 5 \times (0 + 3.6) + \frac{1}{2} \times 5 \times (3.6 + 5.4) + \frac{1}{2} \times 5 \times (5.4 + 6.4) + \frac{1}{2} \times 5 \times \\ &\quad (6.4 + 7.0) \\ &= 9 + 22.5 + 29.5 + 33.5 = 94.5 \text{ m} \end{aligned}$$

$$\begin{aligned} &\text{(Equivalently } \frac{1}{2} \times 5 \times (0 + 2 \times 3.6 + 2 \times 5.4 + 2 \times 6.4 + 7.0) = \\ &2.5 \times 37.8 = 94.5.) \end{aligned}$$

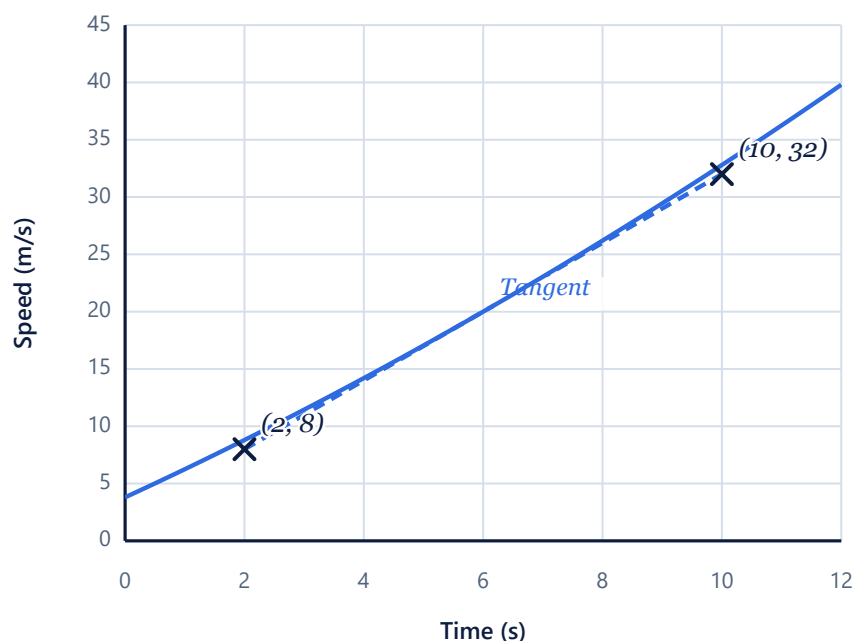
b) The graph is a curve that gets less steep (concave), so each chord lies below the curve and each trapezium misses some of the area. The estimate is an underestimate

5

The speed-time graph for a car is a smooth curve for $0 \leq t \leq 12$, where t is the time in seconds and the speed is in m/s.

A tangent drawn by eye to the curve at $t = 6$ passes through the points $(2, 8)$ and $(10, 32)$.

- Use the tangent to estimate the acceleration of the car at $t = 6$.
- Explain why your answer is only an estimate.



(4 marks)

a) Acceleration at $t = 6$ is the gradient of the tangent at $t = 6$

$$\text{Gradient} = \frac{32 - 8}{10 - 2} = \frac{24}{8} = 3$$

So the acceleration is approximately 3 m/s^2

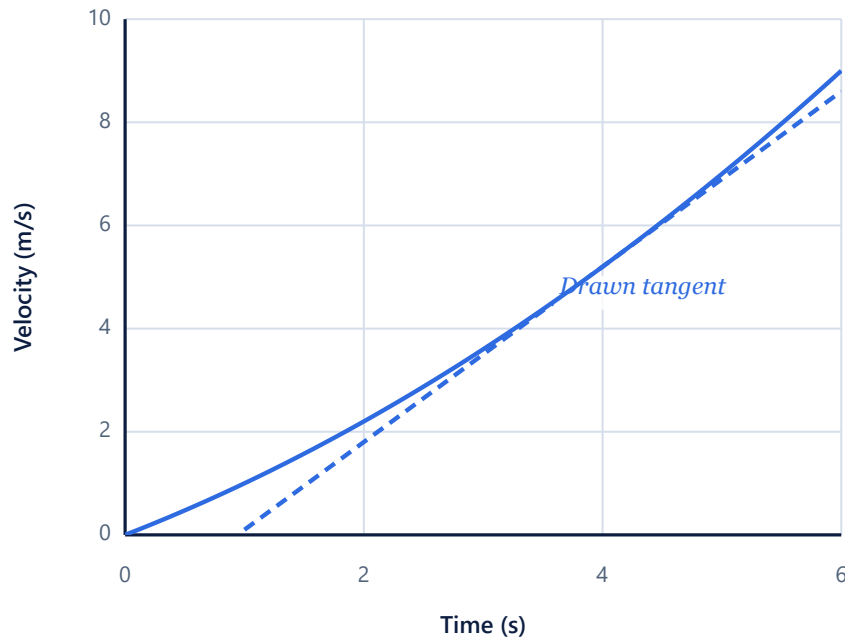
b) The tangent is drawn by eye to a curved graph, so its gradient (and therefore the acceleration) is only an estimate

6

A ball rolls down a slope. Its velocity-time graph is a smooth curve. The table gives the velocity v m/s at time t seconds.

t : 0, 2, 4, 6 v : 0, 2.2, 5.2, 9.0

- A tangent drawn to the curve at $t = 4$ passes through the points (1, 0.1) and (6, 8.6). Use the tangent to estimate the acceleration of the ball at $t = 4$.
- Use the trapezium rule with 3 strips of equal width to estimate the distance the ball travels in the 6 seconds.



(5 marks)

a) Acceleration at $t = 4$ is the gradient of the tangent: $(8.6 - 0.1) \div (6 - 1) = 8.5 \div 5 = 1.7 \text{ m/s}^2$

b) Trapezium strips of width 2:

$$\frac{1}{2} \times 2 \times (0 + 2.2) = 2.2$$

$$\frac{1}{2} \times 2 \times (2.2 + 5.2) = 7.4$$

$$\frac{1}{2} \times 2 \times (5.2 + 9.0) = 14.2$$

$$\text{Estimate} = 2.2 + 7.4 + 14.2 = 23.8 \text{ m}$$

7

Water flows out of a tank. The rate of flow, r litres per minute, is measured every 2 minutes. The graph of r against time t is a smooth curve that falls quickly at first and then flattens out.

t (minutes)	0	2	4	6	8
r (litres/min)	12	8.5	6	4.5	4

- Use the trapezium rule with 4 strips to estimate the area under the graph.
- What does this area tell you about the tank?
- Is your answer to part (a) an over-estimate or an under-estimate? Give a reason.

(4 marks)

a)

width $h = 2$

$$\begin{aligned}
 \text{area} &= \frac{2}{2} \times (12 + 4 + 2 \times (8.5 + 6 + 4.5)) \\
 &= 16 + 2 \times 19 \\
 &= 54
 \end{aligned}$$

b)

rate $\times$ time = volume

litres flowed out in 8 minutes

c)

curve bends below the straight tops

trapezia are too big

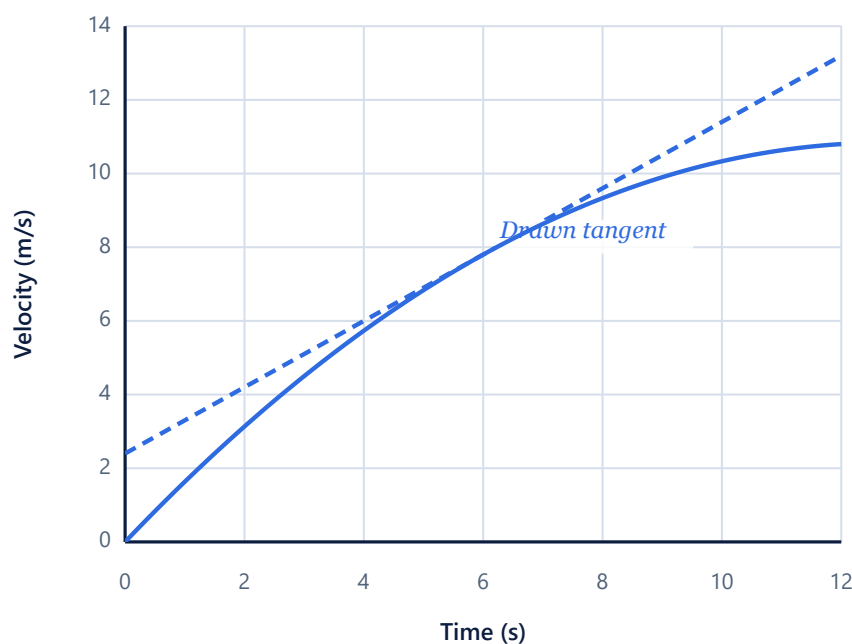
a) 54 b) litres flowed out c) over-estimate

8

A drone accelerates from rest. Its velocity-time graph is a smooth curve that gets less steep throughout the first 12 seconds. The table gives the velocity v m/s at time t seconds.

t : 0, 3, 6, 9, 12 v : 0, 4.5, 7.8, 9.9, 10.8

- Use the trapezium rule with 4 strips of equal width to estimate the distance travelled by the drone in the 12 seconds.
- A tangent drawn to the curve at $t = 6$ passes through the points (0, 2.4) and (12, 13.2). Use the tangent to estimate the acceleration of the drone at $t = 6$.
- Is your answer to part (a) an underestimate or an overestimate of the actual distance? Give a reason.



(7 marks)

a) Trapezium strips of width 3:

$$\frac{1}{2} \times 3 \times (0 + 4.5) = 6.75$$

$$\frac{1}{2} \times 3 \times (4.5 + 7.8) = 18.45$$

$$\frac{1}{2} \times 3 \times (7.8 + 9.9) = 26.55$$

$$\frac{1}{2} \times 3 \times (9.9 + 10.8) = 31.05$$

$$\text{Estimate} = 6.75 + 18.45 + 26.55 + 31.05 = 82.8 \text{ m}$$

b) Gradient of the tangent = $(13.2 - 2.4) \div (12 - 0) = 10.8 \div 12 = 0.9$, so the acceleration at $t = 6$ is about 0.9 m/s^2

c) The question states that the curve gets less steep throughout the interval. Each straight top edge of a trapezium lies below the curve, so the trapezia miss some area: the estimate is an underestimate. The table values alone would not prove this shape between the recorded times